Financial Stability, Growth and Macroprudential Policy[*]

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Abstract

Many emerging market economies have used macroprudential policy to mitigate the risk of financial crises and the resulting output losses. However, macroprudential policy may reduce economic growth in good times. This paper introduces endogenous growth into a small open economy model with occasionally binding collateral constraints in order to study the impact of macroprudential policy on financial stability and growth. In a calibrated version of the model, I find that optimal macroprudential policy reduces the probability of crisis by two thirds at the cost of lowering average growth by a small amount (0.01 percentage point). Moreover, macroprudential policy can generate welfare gains equivalent to a 0.06 percent permanent increase in annual consumption.

Keywords: Macroprudential Policy, Financial Crises, Endogenous Growth
JEL Classification: E44, F38, F43, G38, O30

1 Introduction

In the wake of the Global Financial Crisis in 2008-2009, the use of macroprudential policy to manage boom-bust cycles came to the forefront of macroeconomic research. By limiting excessive capital inflows, the goal of macroprudential policy is to mitigate the risk of financial crises and the resulting output losses. However, policy interventions designed to reduce financial instability may negatively affect long-run economic growth.[1] This raises the question of how much impact macroprudential policy has on growth and whether such impact changes the benefits of macroprudential policy.

To answer these questions, this paper introduces endogenous growth into a small open economy (SOE) model with occasionally binding collateral constraints that has been widely used in the literature to make the case for macroprudential policy. Although previous research looks at the welfare consequence of macroprudential policy, financial crises in the existing framework only have a temporary effect on output,[2] which is inconsistent with the data.[3] By introducing endogenous growth, crises in my model have persistent output-level effects, which allows me to analyze the impact of optimal policy on financial stability and economic growth.

In my model, I endogenize growth by introducing an endogenous productivity process, which can be affected by the occasionally binding collateral constraints. In each period, private agents can use resources to invest in a technology that increases productivity. In a crisis, when the collateral constraint binds, they are forced to cut spending and thus investment in the technology. As a result, crisis periods lead to lower growth.

Unsurprisingly, there is room in my model for policy intervention to address overborrowing. Like other papers (e.g., Jeanne and Korinek (2010b) and Bianchi and Mendoza (forthcoming)), I analyze the role of macroprudential policy by considering a social planner with an instrument to manage capital flows.[4] Unlike the existing literature, however, I do so in an environment that allows me to evaluate the policy's impact on average growth. As an extension, I also analyze the role of a stimulus policy in addition to macroprudential policy by considering a social planner using two instruments to influence

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[1]Some previous literature suggests that countries with more financial crises have higher average growth rates (see Rancière et al. (2008)). Therefore, macroprudential policy aiming to reduce the frequency of crises may lower average growth.

[2]In the existing literature, productivity growth is by assumption exogenous. See Jeanne and Korinek (2010b), Bianchi (2011), Benigno et al. (2013), and Bianchi and Mendoza (forthcoming).

[3]There is strong evidence that financial crises have very persistent effects on output. See Cerra and Saxena (2008), Reinhart and Reinhart (2009), Rogoff and Reinhart (2009), and Ball (2014).

[4]This policy is prudential capital control. See Korinek (2011), Jeanne (2012), Jeanne et al. (2012), and IMF (2012) for a detailed overview.

the composition of spending. This allows me to evaluate the policy debate on ex-ante and ex-post intervention (see Benigno et al. (2013, 2016) and Jeanne and Korinek (2013)).

In general, the impact of macroprudential policy on average growth is ambiguous. On the one hand, macroprudential policy increases growth during crises because it reduces financial vulnerabilities. On the other hand, it also lowers growth during normal periods because it reduces external borrowing and thus the expenditures to increase productivity. The calibrated version of my model reveals that optimal macroprudential policy reduces the probability of crises from 6.2 percent to 1.9 percent (about two thirds), at the cost of lowering average growth by 0.01 percentage point.

Furthermore, I find that the welfare gains from optimal macroprudential policy are equivalent to a 0.06 percent permanent increase in annual consumption. Similar to existing literature, macroprudential policy increases welfare by limiting the likelihood of financial crises, therefore helping agents to smooth consumption. In fact, in the model, that effect is stronger with endogenous growth. However, macroprudential policy successfully restricts over-borrowing in the upswing, thus reducing average growth. Overall, macroprudential policy still improves welfare. The gains are similar to those in the related literature (see Jeanne and Korinek (2010b) and Bianchi (2011)).

In my model, the use of macroprudential policy limits borrowing and thus spending for the technology for growth. A natural question, then, is to ask whether there are other policy tools that can be implemented in tandem with macroprudential policy (a capital flow tax) to offset the negative impact of the policy on growth. To answer this question, I consider a social planner with two instruments. The first instrument is a capital flow tax, while the second instrument is a growth subsidy that can be used to change the composition of spending on the technology for growth. This exercise also allows me to analyze the role of ex-post intervention because this social planner uses two tools to intervene both ex-ante and ex-post, different from the social planner with only the capital flow tax who only intervenes ex-ante.

I find that the social planner with two instruments can generate much larger welfare benefits than the social planner with only one instrument. Quantitatively, the gains are equivalent to a 0.24 percent permanent increase in annual consumption. Two instruments enable the social planner to intervene ex-post and thus mitigate the cost of crises. These two instruments used ex-post act as a stimulus policy. Ex-ante, the social planner uses capital flow tax to correct over-borrowing in the credit market. In this case, capital flow tax act as a macroprudential policy. However, the social planner also uses the growth subsidy to offset the negative effect of macroprudential policy on growth. The ex-ante

growth subsidy thus belongs to the stimulus policy. The availability of the stimulus policy is beneficial because it leads to a short-run boom in both growth and consumption, which is not seen in the economy with only macroprudential policy.

Relation to Literature

This paper is related to the literature on the relationship between growth and stability, in which empirical evidence often leads to mixed results. There are papers on the cross-country relationship between average growth and volatility of growth. For example, Ramey and Ramey (1995) find a negative relationship between average growth and volatility of growth, while Rancière et al. (2008) argue that countries experiencing more crises (more volatile growth) have higher average growth (see Levine (2005) for a summary). Moreover, there are also papers on the impact of policy on growth and financial stability. For example, Sánchez and Gori (2016) find that certain growth-promoting policies can have negative side-effects on financial stability, while Boar et al. (2017) find that macroprudential policy can increase both financial stability and long-run economic growth. This paper finds a negative relationship between average growth and financial stability for macroprudential policy, consistent with Rancière et al. (2008) and Sánchez and Gori (2016). However, this relationship depends on calibrations and might become positive in some cases, which is consistent with the findings in Ramey and Ramey (1995) and Boar et al. (2017).

This paper is also related to the literature on short-run fluctuations and growth. There are two existing approaches in the literature to introduce endogenous growth into a standard DSGE framework: One approach models growth following Romer (1990), such as Comin and Gertler (2006), Queraltó (2015), and Guerron-Quintana and Jinnai (2014). The other approach models growth following Aghion and Howitt (1992), such as Ates and Saffie (2016) and Benigno and Fornaro (2017). My way of modeling growth is similar to the first approach, which preserves the representative-agent framework. However, unlike the existing literature, which focuses on a positive analysis, my paper is interested in the characterization of optimal policy and the policy's impact on growth and welfare.

Finally, this paper belongs to the literature on optimal macroprudential policy and capital flow management. The theoretical rationale for macroprudential policy includes pecuniary externalities (see Lorenzoni (2008), Jeanne and Korinek (2010a), and Dávila and Korinek (2017)) and aggregate demand externalities (see Farhi and Werning (2016) and Korinek and Simsek (2016)). The general takeaway from the theories is that ex-ante policy intervention can be welfare-improving, since it addresses over-borrowing in the

credit market and thus reduces financial instability. However, the literature has been silent on the effect of ex-ante intervention on economic growth, which is the main focus of this paper. Specifically, this paper introduces endogenous growth into a standard SOE-DSGE model with occasional binding constraints (see Jeanne and Korinek (2010b), Mendoza (2010), and Bianchi (2011),). Unlike in other literature, crises have persistent output-level effects in this model, consistent with the empirical evidence. Furthermore, endogenous growth also enables me to evaluate the debate on ex-ante versus ex-post interventions (see Benigno et al. (2013, 2016) and Jeanne and Korinek (2013)). In particular, I focus not only on the benefits of ex-ante and ex-post interventions but also on their impacts on economic growth.

The organization of this paper is as follows: Section 2 presents a benchmark model; Section 3 presents a normative analysis for macroprudential policy; Section 4 presents the calibration procedure and model performance; Section 5 presents quantitative analysis; Section 6 presents an extension to analyze the role of other policy instruments; and Section 7 concludes.

2   Model Economy

This section introduces an analytical framework that incorporates endogenous growth into an SOE model as in Jeanne and Korinek (2010b) and Bianchi and Mendoza (forthcoming). One feature of the model is an occasionally binding collateral constraint. It has been used in the literature since Mendoza (2010) to model financial crises. In the model, normal periods are when the constraint is slack, and crisis periods are when the constraint binds.

2.1   Analytical Framework

In my model, the economy is populated by a continuum of identical households that have access to an international capital market and a technology that increases productivity. Due to friction in the financial market, there exist collateral borrowing constraints, and the maximum amount of external borrowing cannot exceed the value of collateral. In normal periods, when the constraints are slack, households are able to finance their desired levels of expenditure through external borrowing. The economy thus grows at a normal rate. In crises, when the collateral constraints bind, households cannot finance enough expenditures for the technology. As a result, the growth rate drops.

Preferences: Households have the following Constant Relative Risk Aversion (CRRA) preferences with the Stone-Geary functional form (see Geary (1950) and Stone (1954)):

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t - \mathcal{H}_t\right) \equiv E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t - \mathcal{H}_t\right)^{1-\gamma}}{1-\gamma} \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, γ is the coefficient of risk aversion, c_t is consumption, and $\mathcal{H}_t$ is the subsistence level of consumption. Given that the economy is growing, I assume that $\mathcal{H}_t$ depends on the level of endogenous productivity z_t and takes the functional form:[5]

$$\mathcal{H}_t = h z_t \tag{2}$$

Without $\mathcal{H}_t$, private agents find it costly to cut z_{t+1}, since that implies a permanent future loss in output.[6] As a result, the growth rate barely falls when there is a negative shock. The presence of $\mathcal{H}_t$ reduces the cost of cutting z_{t+1}, since the future subsistence level of consumption $\mathcal{H}_{t+1}$ decreases with z_{t+1}. Therefore, this non-standard assumption with $\mathcal{H}_t$ allows my model to generate a large growth rate decrease in financial crises.[7]

Production Function: Production only requires a productive asset n_t as an input and takes the following form:

$$y_t = A_t n_t^{\alpha} \tag{3}$$

where A_t represents the productivity level in the economy and $\alpha \in (0,1)$. Productive asset n_t is an endowment to households and is normalized to 1. It corresponds to an asset in fixed supply, such as land. In each period, households trade the productive asset n_t at a market-determined price q_t.

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[5]$h > 0$ is a constant.

[6]As I will explain below, future output y_{t+1} depends on productivity z_{t+1}.

[7]In a model with endogenous growth, it is very costly to reduce productivity, and thus growth, following a shock. Instead, private agents cut consumption spending. To have a large decrease in growth, one may want to raise the cost of cutting consumption, such as by increasing the risk-aversion of utility functions. However, neither a high coefficient of risk aversion γ nor Epstein-Zin preference leads to a large decrease in growth following a crisis. One might also want to introduce habit, as in Campbell and Cochrane (1999). But their formulation introduces an additional state variable, which complicates the computation. My way of modeling $\mathcal{H}_t$ is simpler, and one can interpret it as a habit that depends on the level of z_t in the economy.

Endogenous Productivity: The level of productivity A_t takes the following form:

$$A_t = \theta_t z_t \tag{4}$$

where θ_t is a stationary exogenous productivity shock, and z_t is non-stationary endogenous productivity chosen by private agents.

Source of Growth: Growth in the economy comes from the endogenous productivity z_t that households can choose. Specifically, there is a technology that costs $\Psi(z_{t+1}, z_t)$ units of consumption to elevate endogenous productivity from z_t to z_{t+1}. I call $\Psi(z_{t+1}, z_t)$ "growth-enhancing expenditures," which include all the expenditures that facilitate long-term economic growth. Here I do not take a stand on any particular form of endogenous growth, but use a generic form that includes many models in the growth literature. For example, $\Psi(z_{t+1}, z_t)$ includes physical capital investment in the AK growth framework as in Romer (1986), human capital investment as in Lucas (1988), R&D expenditure as in Romer (1990) and Aghion and Howitt (1992), etc. The only restriction is that there are no externalities in the process of choosing z_{t+1}. When private agents choose z_{t+1}, they internalize its impact on not only the future subsistence level of consumption $\mathcal{H}_{t+1}$ but also the future cost function, $\Psi(z_{t+2}, z_{t+1})$. This restriction thus shuts down any externalities in endogenous growth.[8] This departs from the literature on short-run fluctuations and growth, where economic growth is typically suboptimal (see Comin and Gertler (2006) and Kung and Schmid (2015)).

Financial Friction: I introduce a collateral constraint on external borrowing following Jeanne and Korinek (2010b) and Bianchi and Mendoza (forthcoming). Specifically, households can purchase b_{t+1} units of a one-period bond from the international market in each period, and these bonds promise a gross interest rate $1 + r$ in the next period. The domestic economy is atomistic in the international world and takes the interest rate as given. Furthermore, bonds are supplied with infinite elasticity. However, there is a source of financial friction in the market: Private agents need to post their productive assets as collateral for external borrowing, and the maximum amount of external borrowing cannot exceed a fraction $\phi \in (0, 1)$ of the collateral value q_t.[9] Therefore, the collateral constraint

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[8]As I will explain in the next section, there are pecuniary externalities in the economy that justify an optimal policy. However, both externalities in growth and pecuniary externalities typically call for policy intervention to increase national saving. If both of them present in the economy, it is hard to disentangle their effects. Furthermore, externalities in endogenous growth tend to dominate pecuniary externalities.

[9]One rationale for the collateral constraint is as follows: There is a moral hazard problem between

can be written as

$$-b_{t+1} \leq \phi q_t \tag{5}$$

Budget Constraint: In each period, households make expenditure plans for consumption c_t and growth-enhancing expenditures $\Psi(z_{t+1}, z_t)$ and purchase productive assets $q_t n_{t+1}$ and bond holdings b_{t+1}. Their incomes come from the output y_t, sale of productive assets $q_t n_t$, and existing bond holdings $(1+r)b_t$. As a result, the budget constraint can be written as follows:

$$c_t + \Psi(z_{t+1}, z_t) + q_t n_{t+1} + b_{t+1} = y_t + q_t n_t + (1+r) b_t, \tag{6}$$

Market Clearing: There are two markets in the economy: the final goods market and the productive asset market. Given that the productive asset is in fixed supply and owned by the households, the equilibrium condition implies that

$$n_t = 1, \ \forall t \tag{7}$$

The final goods market can be pinned down by aggregating the budget constraint for each household and applying the equilibrium condition (7) in the productive asset market.

$$c_t + \Psi(z_{t+1}, z_t) + b_{t+1} = y_t + (1+r) b_t, \tag{8}$$

2.2   Competitive Equilibrium (CE)

Competitive Equilibrium: In this economy, equilibrium consists of a stochastic process $\{c_t, z_{t+1}, n_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ chosen by the households and an asset price $\{q_t\}_{t=0}^{\infty}$, given initial values $\{b_0, z_0\}$ and the exogenous shock $\{\theta_t\}_{t=0}^{\infty}$ such that utility (1) is maximized, constraints (5) and (6) are satisfied, and the productive assets and goods market clear, i.e., conditions (7) and (8) are satisfied.

Recursive Formulation: It is convenient to define net consumption by $c_t^h = c_t - \mathcal{H}_t$ and write the problem in a recursive formulation. State variables at time t include the en-

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domestic households and international investors (see Jeanne and Korinek (2010b)). Households have the option to invest in a scam that prevents international investors from seizing future productive assets. This implies that households can default on their debts without any punishment. The investors, however, cannot coordinate to punish the households by excluding them from the market. What they can do is take households to court before the scam is completed. By doing so, they can only seize a fraction ϕ of productive assets and sell them to other households at the prevailing market price q_t. As a result, rational international investors will restrict the amount of external borrowing up to ϕq_t.

dogenous variables $\{z_t, n_t, b_t\}$ and the exogenous variable θ_t. I can write the optimization problem as follows:

$$V_t^{CE}(z_t, n_t, b_t, \theta_t) = \max_{c_t^h, z_{t+1}, n_{t+1}, b_{t+1}} u\left(c_t^h\right) + \beta E\left[V_{t+1}^{CE}(z_{t+1}, n_{t+1}, b_{t+1}, \theta_{t+1})\right]$$

$$\text{s.t.} \quad c_t^h + hz_t + \Psi(z_{t+1}, z_t) + q_t n_{t+1} + b_{t+1} = \theta_t z_t n_t^\alpha + q_t n_t + (1+r)b_t,$$

$$-b_{t+1} \leq \phi q_t.$$

The maximization problem yields the following optimality conditions for each period:

$$\lambda_t^{CE} = u'\left(c_t^h\right) \tag{9}$$

$$\lambda_t^{CE} \Psi_{1,t} = \beta E_t\left[\lambda_{t+1}^{CE}(\theta_{t+1} - h - \Psi_{2,t+1})\right] \tag{10}$$

$$\lambda_t^{CE} q_t = \beta E_t\left[\lambda_{t+1}^{CE}(\alpha\theta_{t+1}z_{t+1} + q_{t+1})\right] \tag{11}$$

$$\lambda_t^{CE} = \mu_t^{CE} + \beta(1+r)E_t\left[\lambda_{t+1}^{CE}\right] \tag{12}$$

where $\Psi_{1,t} = \frac{\partial \Psi(z_{t+1}, z_t)}{\partial z_{t+1}}$ and $\Psi_{2,t+1} = \frac{\partial \Psi(z_{t+2}, z_{t+1})}{\partial z_{t+1}}$. λ_t^{CE} and μ_t^{CE} are Lagrangian multipliers associated with the budget constraint and collateral constraint, respectively.

Condition (9) is the marginal valuation of wealth for households. Condition (10) is the key equation for growth in this model, where private agents equate the marginal cost of choosing z_{t+1} with the marginal benefit. The cost is reflected in the partial derivative of the technology function $\Psi_{1,t}$, while the benefit includes a future output θ_{t+1}, excluding the normalized future subsistence level of consumption, h and the partial derivative of future technology function, $\Psi_{2,t+1}$. The marginal cost and marginal benefit are evaluated at the marginal valuation of wealth in periods t and $t+1$ respectively. The third condition (11) is a standard asset pricing function, where holding productive asset n_{t+1} yields a dividend income $\alpha\theta_{t+1}z_{t+1}$ and capital gains q_{t+1}. The last condition (12) is the Euler equation for holding bonds. The additional term μ_t^{CE} captures the effect of collateral constraint on the external borrowing. When the collateral constraint (5) binds, the marginal benefit of borrowing to increase consumption exceeds the expected marginal cost by an amount equal to the shadow price of relaxing collateral constraint μ_t^{CE}.

Normalized Economy: To solve for a stationary equilibrium, I normalize all the endogenous variables by z_t and denote this by variables with hats. Specifically, I denote $\hat{x}_t = \frac{x_t}{z_t}$, where $x_t = \{c_t^h, b_t, q_t, V_t^{CE}, \cdots\}$, and endogenous growth rate $g_{t+1} = \frac{z_{t+1}}{z_t}$. The normalized equilibrium conditions are given in Appendix C.

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3   Optimal Macroprudential Policy

Consistent with the literature, there is a role for macroprudential policy in the economy due to the presence of pecuniary externalities (see Lorenzoni (2008) and Dávila and Korinek (2017)).[10] These pecuniary externalities are related to a vicious cycle associated with the collateral borrowing constraints. Intuitively, private agents need to cut spending when a negative shock hits and the constraints bind. However, asset prices fall with a decline in spending, and private agents need to cut spending further due to lower collateral values and tighter borrowing constraints. Therefore, the initial shock is endogenously amplified through the constraints. Importantly, private agents, taking the asset price as given, fail to internalize their contributions to this vicious cycle, which represents pecuniary externalities in the economy. As a result, they over-borrow in normal periods. The optimal macroprudential policy is designed to correct this over-borrowing in the credit market.

Following the literature, I first define the social planner's problem and then choose macroprudential policy to implement the allocation (see Jeanne and Korinek (2010b), Bianchi (2011), and Bianchi and Mendoza (forthcoming)). This is similar to the "primal approach" in optimal policy analysis (originally from Stiglitz (1982)), in which the social planner can choose allocations subject to resource, implementability, and collateral constraints. This formulation allows me to see the wedge between the social planner and private agents in choosing allocations and understand the inefficiencies in the economy. To implement the social planner's allocation, I consider what tax or subsidy with lump-sum transfers is needed to close the wedge. In this case, a tax on capital flows is needed.

Specifically, I consider the social planner who chooses allocations on behalf of the representative household subject to the same constraints as private agents, but who lacks the ability to commit to future policies. Importantly, I assume that the asset price q_t remains market-determined and that the Euler equation of asset price (11) enters the social planner's problem as an implementability constraint. The implicit rationale is that the social planner cannot directly intervene with respect to the asset price but internalizes how the allocations affect it and thus the collateral constraint.[11]

Furthermore, I assume that endogenous productivity z_{t+1} is chosen by private agents

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[10]Pecuniary externalities refer to externalities associated with prices. In an economy with incomplete markets, allocations with pecuniary externalities are generically sub-optimal. For a detailed proof, see early contributions by Geanakoplos and Polemarchakis (1986) and Greenwald and Stiglitz (1986).

[11]I do not allow the social planner to trade assets on behalf of private agents. One rationale is that private agents are better than the planner at observing fundamental payoffs of financial assets (see Jeanne and Korinek (2010b)).

and that the Euler equation of productivity (10) also enters the social planner's problem as an additional implementability constraint. This is because I use macroprudential policy to decentralize this social planner's allocation and the policy is designed to correct the wedge only in the bond holdings. To correct other wedges, such as that in productivity, an additional instrument is needed. I analyze this case in Section 6.

I call the social planner with macroprudential policy a *macroprudential social planner* and denote her allocation with a superscript "MP". As described before, the maximization problem can be written as

$$V_t^{MP}(z_t, b_t, \theta_t) = \max_{c_t^h, z_{t+1}, b_{t+1}, q_t} u\left(c_t^h\right) + \beta E\left[V_{t+1}^{MP}(z_{t+1}, b_{t+1}, \theta_{t+1})\right]$$

$$\text{s.t.} \quad c_t^h + h z_t + \Psi(z_{t+1}, z_t) + b_{t+1} = \theta_t z_t + (1+r)b_t,$$

$$-b_{t+1} \leq \phi q_t,$$

$$u'(c_t^h)q_t = \underbrace{\beta E_t\left[u'(c_{t+1}^h)\left(\alpha\theta_{t+1}z_{t+1} + q_{t+1}\right)\right]}_{G(z_{t+1}, b_{t+1})}, \tag{13}$$

$$u'(c_t^h)\Psi_{1,t} = \underbrace{\beta E_t\left[u'(c_{t+1}^h)\left(\theta_{t+1} - h - \Psi_{2,t+1}\right)\right]}_{I(z_{t+1}, b_{t+1})}. \tag{14}$$

where equations (13) and (14) are two implementation constraints, i.e., the Euler equations of choosing productive assets and productivity. I write implementation constraints as functions of future endogenous state variables z_{t+1} and b_{t+1}, since I want to solve for time-consistent policy functions as in Bianchi and Mendoza (forthcoming).

Given the definition of the macroprudential social planner, it is straightforward to define constrained inefficiency as follows:

Definition 1. *Constrained Inefficiency*
The competitive equilibrium displays constrained inefficiency if it differs from the allocation chosen by the macroprudential social planner.

To understand the difference between private agents and the macroprudential social planner, I derive the optimality conditions of MP as follows:

$$\lambda_t^{MP} = u'(c_t^h) - \xi_t^{MP} u''(c_t^h)q_t - \nu_t^{MP} u''(c_t^h)\Psi_{1,t} \tag{15}$$

$$\lambda_t^{MP}\Psi_{1,t} - \xi_t^{MP}G_{1,t} - \nu_t^{MP}\left[I_{1,t} - u'(c_t^h)\Psi_{11,t}\right]$$
$$= \beta E_t\left[\lambda_{t+1}^{MP}\left(\theta_{t+1} - h - \Psi_{2,t+1}\right) - \nu_{t+1}^{MP}u'(c_{t+1}^h)\Psi_{12,t+1}\right] \tag{16}$$

$$\phi\mu_t^{MP} = \xi_t^{MP}u'(c_t^h) \tag{17}$$

$$\lambda_t^{MP} = \mu_t^{MP} + \xi_t^{MP}G_{2,t} + \nu_t^{MP}I_{2,t} + \beta(1+r)E_t\left[\lambda_{t+1}^{MP}\right] \tag{18}$$

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where $\Psi_{11,t} = \frac{\partial^2 \Psi(z_{t+1}, z_t)}{\partial z_{t+1}^2}$, $\Psi_{12,t+1} = \frac{\partial^2 \Psi(z_{t+2}, z_{t+1})}{\partial z_{t+2} \partial z_{t+1}}$, $G_{1,t} = \frac{\partial G(z_{t+1}, b_{t+1})}{\partial z_{t+1}}$, $G_{2,t} = \frac{\partial G(z_{t+1}, b_{t+1})}{\partial b_{t+1}}$, $I_{1,t} = \frac{\partial I(z_{t+1}, b_{t+1})}{\partial z_{t+1}}$, and $I_{2,t} = \frac{\partial I(z_{t+1}, b_{t+1})}{\partial b_{t+1}}$. λ_t^{MP}, μ_t^{MP}, ξ_t^{MP}, and ν_t^{MP} are Lagrangian multipliers associated with the budget constraint, collateral constraint, and two implementation constraints, respectively.

Wedge in Marginal Valuation of Wealth: The main difference between CE and MP is reflected in the marginal valuation of wealth, λ_t^{CE} and λ_t^{MP}. One can see that the wedge includes two terms due to the presence of implementation constraints: The first term is $-\xi_t^{MP} u''(c_t^h) q_t$, which captures pecuniary externalities in the economy, and the second term is $-\nu_t^{MP} u''(c_t^h) \Psi_{1,t}$, which captures the inability of the social planner to change z_{t+1}. Consistent with results in the literature, the first term is positive due to condition (17). Uniquely, I also have the second term with ν_t^{MP}, which is the shadow price of implementation constraint (14). The value of ν_t^{MP} is given by the optimality condition (16). Quantitatively, it is small. Hence, the wedge $-\xi_t^{MP} u''(c_t^h) q_t - \nu_t^{MP} u''(c_t^h) \Psi_{1,t}$ is positive.

Due to this wedge, the competitive equilibrium is constrained inefficient, and the social planner chooses a different allocation than do private agents. However, the difference appears only when the constraint is slack. The reason is that the social planner cannot change the allocation when the constraint binds. In the period when the collateral constraint is slack, i.e., $\mu_t^{MP} = 0$, the social planner chooses a higher level of bond holding than do private agents due to a higher valuation of future wealth $E_t \left[\lambda_{t+1}^{MP} \right]$ (see the optimality conditions of bond holding in CE and MP, (12) and (18)).[12] Hence, there is an over-borrowing issue in competitive equilibrium, consistent with the literature.

Implementation: I assume that the planner has access to a macroprudential tax $\tau_t^{MP,b}$ on capital flows and a lump-sum transfer T_t^{MP}. The budget constraint for private agents becomes

$$c_t^h + hz_t + \Psi(z_{t+1}, z_t) + q_t n_{t+1} + \left(1 - \tau_t^{MP,b}\right) b_{t+1} = y_t + q_t n_t + (1+r) b_t + T_t^{MP}$$

where $T_t^{MP} = -\tau_t^{MP,b} b_{t+1}$.

Proposition 1. *Decentralization with Macroprudential Policy*
The macroprudential social planner's allocation can be implemented by a macroprudential tax $\tau_t^{MP,b}$ on capital flows that is rebated to private agents with a lump-sum transfer T_t^{MP}.

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[12]Quantitatively, the term $\nu_t^{MP} u''(c_t^h) \Psi_{1,t} + \nu_t^{MP} I_{2,t}$ is small.

Furthermore, the tax $\tau_t^{MP,b}$ *is given by*

$$
\tau_t^{MP,b} = \frac{\beta g_{t+1}^{-\gamma} (1+r) E_t \left[\gamma \phi \hat{\mu}_{t+1}^{MP} \hat{q}_{t+1} \left(\hat{c}_{t+1}^h \right)^{-1} + \gamma \hat{\nu}_{t+1}^{MP} \left(\hat{c}_{t+1}^h \right)^{-\gamma-1} \Psi_{1,t+1} \right]}{\left(\hat{c}_t^h \right)^{-\gamma}}
$$

$$
- \frac{\gamma \phi \hat{\mu}_t^{MP} \hat{q}_t \left(\hat{c}_t^h \right)^{-1} + \gamma \hat{\nu}_t^{MP} \left(\hat{c}_t^h \right)^{-\gamma-1} \Psi_{1,t} - \phi \hat{\mu}_t^{MP} g_{t+1}^{-\gamma} \hat{G}_{2,t} \left(\hat{c}_t^h \right)^{\gamma} - \hat{\nu}_t^{MP} g_{t+1}^{-1-\gamma} \hat{I}_{2,t}}{\left(\hat{c}_t^h \right)^{-\gamma}}
$$

Proof. See Appendix D.1.

Consistent with the literature, a macroprudential tax $\tau_t^{MP,b}$ is used to correct the wedge between λ_t^{MP} and λ_t^{CE}. It is positive in the quantitative exercise, since the Lagrangian multiplier ν_t^{MP} is small. Hence macroprudential policy is also used to correct the over-borrowing issue in the economy.

4   Calibration

This section first describes an 11-year event window that the model targets. It then shows parameter values and the model's ability to fit the data.

4.1   Targeted Event Window

One key feature of the model is its generation of such persistent output-level effects of financial crises as found in the data (see Cerra and Saxena (2008), Rogoff and Reinhart (2009), and Ball (2014)). To quantify the magnitude of output cost for later calibration, I construct an 11-year event window of output growth rates centering on one specific type of financial crisis in emerging markets, i.e., sudden stop episodes.[13] These episodes occur when there is a sudden slowdown in private capital inflows to emerging market economies and a corresponding sharp reversal in current account balances. For the identification of sudden stops, I use the episodes in Calvo et al. (2006) ("Calvo episodes"), whose criterion is based on a sharp reversal in current account balances and a spike in spreads. For robustness, I also use episodes identified in Korinek and Mendoza (2014) ("KM episodes") and report the results in Appendix B.

The left panel of Figure 1 shows that the growth rate of real GDP per capita is a stationary process and falls to -5.65 percent at the time of crises. I also construct an

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[13]The source of real GDP per capita is explained in Appendix A.

event window for "Total Factor Productivity (TFP)" in the right panel of Figure 1 and find that productivity displays a similar pattern to output, consistent with the predictions of my model.

Figure 1: Growth Rates in Sudden Stop Episodes (%)
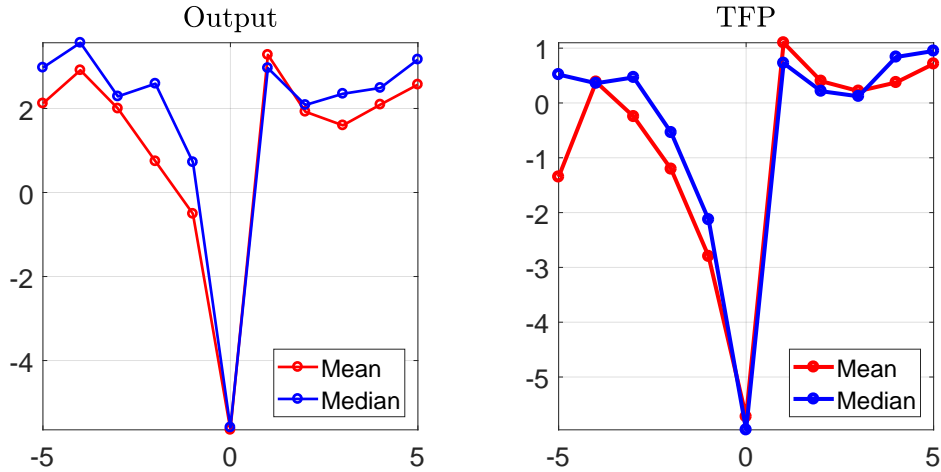
Note: The series are constructed using an 11-year window centering on the sudden stop episodes.

4.2 Parameter Values

I calibrate the model to annual frequency using 55 countries' data from between 1961 and 2015 (see Appendix A for details). The model can be solved using a variant of the endogenous gridpoint method, as in Carroll (2006) (see Appendix F for details). There is only one shock in the economy: the exogenous technology shock θ_t, which follows the process below. I discretize the process using Rouwenhorst method as in Kopecky and Suen (2010).

$$\log \theta_t = \rho \log \theta_{t-1} + \varepsilon_t, \text{ where } \varepsilon_t \sim N(0, \sigma^2)$$

where ρ and σ are persistence and volatility of the shock, and ε_t is a random variable following a normal distribution.

It is important to have the shock θ_t in the model to capture the fall of output growth during crises, as seen in Figure 1. Without a fall in θ_t, one cannot explain the negative output growth rate in crises, since output y_t depends on the predetermined productivity z_t and the exogenous productivity θ_t.[14] Furthermore, the endogenous response of produc-

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[14]Admittedly, other shocks, such as financial shocks and interest rate shocks, are important for under-

tivity z_{t+1} prevents the output growth rate after crises from being higher than its long-run average, consistent with the event window.[15]

Assumption 1. *Cost function* $\Psi(z_{t+1}, z_t)$ *is quadratic and takes the following form:*

$$\Psi(z_{t+1}, z_t) = \left[\left(\frac{z_{t+1}}{z_t} - \psi \right) + \kappa \left(\frac{z_{t+1}}{z_t} - \psi \right)^2 \right] z_t,$$

where $\psi > 0$ *and* $\frac{z_{t+1}}{z_t} \geq \psi$.

I impose a simple quadratic form on $\Psi(z_{t+1}, z_t)$ so as to calibrate my model. Given that this way of modeling growth is generic, I calibrate the parameter values in the function by reference to some moments in the data. For example, κ is a scale parameter and is used to match the average share of consumption in GDP. The parameter ψ is the minimum level of endogenous growth g_{t+1} in the model and is used to match the output growth rate after crises in the targeted event window.

I need to assign values to 10 parameters in the model: $\{\beta, r, \gamma, h, \psi, \kappa, \alpha, \rho, \sigma, \phi\}$. The calibration proceeds in two steps. First, some parameter values are standard in the literature. For example, I choose the interest rate r to be 6 percent and the coefficient of risk aversion parameter γ to be 2. The parameter α equals productive asset income's share of total income, and I choose 0.2 following Jeanne and Korinek (2010b). Second, given these parameter values, I jointly choose the remaining parameters to match relevant moments in the data and the targeted event window in Figure 1.

Specifically, I use the following parameters to match data moments. Parameter β determines the incentive to borrow and is chosen to match the long-run Net Foreign Asset (NFA) to GDP ratio (-30 percent). Parameter ρ is chosen to match the correlation between the current account and output at -0.25, since I focus on the relationship between capital flows and output growth.[16] Parameter ϕ determines the maximum value of borrowing in the economy and thus the probability of crises.[17] In the model, I define crisis

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standing financial crises. However, these shocks alone cannot lead to a drop of output growth in crises in the model, since the productivity z_t is predetermined.

[15]One could also have an exogenous trend shock, as in Aguiar and Gopinath (2007). Introducing an exogenous trend shock, however, does not allow me to analyze the policy's impact on growth.

[16]Aguiar and Gopinath (2007) find that the persistence of shocks governs the correlation between the current account and output. The correlation is constructed by first de-trending the output series with a HP filter and then calculating the correlation between the current account to GDP ratio and the cyclical component of output.

[17]I calibrate the model such that the collateral constraint marginally binds in the long run and the

episodes as periods when constraints bind and the magnitude of current account reversal exceeds 1 standard deviation of its long-run average (see Bianchi (2011)). The parameter ϕ is chosen to match the probability of crises at 5.5 percent, a standard value in the literature (see Bianchi (2011) and Eichengreen et al. (2008)). Furthermore, parameters h and κ are jointly chosen to match the average growth rate, 2.3 percent, and the share of consumption in GDP, 77.6 percent. Specifically, h and κ have to satisfy the normalized resource constraint (8) and the Euler equation of z_{t+1} (10) as follows:

$$\underbrace{\hat{c}_{ss}}_{77.6\%} + \hat{\Psi}(\underbrace{g_{ss}}_{1+2.3\%}) = 1 + \frac{1 + r - g_{ss}}{g_{ss}} \underbrace{\hat{b}_{ss}g_{ss}}_{-30\%}$$

$$\Psi_1(g_{ss}) = \beta g_{ss}^{-\gamma}(1 - h - \Psi_2(g_{ss}))$$

where the average value of θ_t is normalized at 1, and the value of h and κ depend on the value of β and ψ.[18]

As explained before, I also want to match the event window in Figure 1. The volatility σ governs the minimum level of the exogenous shock θ_t and thus the decline in the output growth rate during crises. Parameter ψ determines the minimum level of the endogenous growth rate g_{t+1} and thus the decline in the output growth rate one year after crises. Therefore, I choose σ and ψ to jointly match the output growth rate during crises (-5.65 percent) and one period after crises (3.28 percent) in the event window.

In sum, given values of $\{r, \gamma, \alpha, \eta\}$, I pick values of $\{\beta, \psi, \rho, \sigma\}$, which determine values of $\{\phi, \kappa, h\}$. I then simulate the model, calculate moments of the simulated data, construct an event window as in Figure 1, and then compare the simulation results with the actual data moments and the targeted event window.[19] The values of all parameters are reported in Table 1.

---

following relationship holds in the steady states:

$$\underbrace{-\hat{b}g_{ss}}_{30\%} = \phi\hat{q}$$

$$\hat{q} = \frac{\beta g_{ss}^{1-\gamma}}{1 - \beta g_{ss}^{1-\gamma}}\alpha$$

[18]Here, I calibrate the economy so that in the long run it is unconstrained and the collateral constraint marginally binds.

[19]In particular, I simulate the model for 11,000 periods and throw away the first 1000 periods. Data moments are calculated based on the remaining 10,000 periods of simulated data. Furthermore, I identify crisis episodes in the simulated data and calculate the average output growth rate during crises and one period after crises.

Table 1: Calibration

	Value	Source/target
Parameter in production function	$\alpha = 0.2$	Jeanne and Korinek (2010b)
Risk-free interest rate	$r = 6\%$	Benigno et al. (2013)
Risk aversion	$\gamma = 2$	Standard in the literature
Volatility of technology shock	$\sigma = 0.04$	Output growth rate at time of crises $= -5.65$ %
Parameter in Ψ functions	$\psi = 0.95$	Output growth rate one year after crises $= 3.28\%$
Parameter in Ψ functions	$\kappa = 26.29$	Consumption-GDP ratio $= 77.6\%$
Subsistence level parameter	$h = 0.51$	Average GDP growth $= 2.3\%$
Discount rate	$\beta = 0.968$	Probability of crisis $= 5.5\%$
Persistence of technology shock	$\rho = 0.83$	Correlation between current account and output $= -0.25$
Collateral constraint parameter	$\phi = 0.0852$	NFA-GDP ratio $= -30\%$

4.3   Model Performance

Table 2 reports model and data moments. One can see that the model is able to match targeted moments in the data. As other models with occasionally binding collateral constraints, crisis episodes are rare events in my model and occur with a probability of 6.2 percent in the simulation.

Table 2: Moments: Data and Model

Targeted Moments	Data	Model
Average GDP growth (%)	2.30	2.31
Probability of crisis (%)	5.50	6.23
NFA-GDP ratio (%)	-30.00	-27.18
Consumption-GDP ratio (%)	77.6	77.53
Correlation between current account and output	-0.25	-0.22

Unlike existing models in the literature, my model can generate the growth rate dynamics in Figure 1. To see this, I simulate the model, identify crisis episodes and construct an 11-period event window for different variables in Figure 2. Not surprisingly, crises occur when there is a large drop in the exogenous shock θ_t. The current account experiences a large reversal because the borrowing constraints bind and private agents have to cut their external borrowing, i.e., an increase in $\hat{b}_{t+1}$. Furthermore, these events are accompanied by a decline in spending such as consumption $\hat{c}_t$ and growth-enhancing expenditures (reflected in a decline in the endogenous growth rate g_{t+1}). The asset price $\hat{q}_t$ also drops, which leads to an amplification effect through collateral constraints. Fortunately, my model captures the empirical regularity of crises. Importantly, it can capture the persistent output-level effects of crises as in the data: Output growth rates fall during crises with a decline in θ_t and only go back to the long-run average level after crises. This occurs

because the endogenous growth rate g_{t+1} decreases during crises.

Figure 2: Event Window: Model and Data
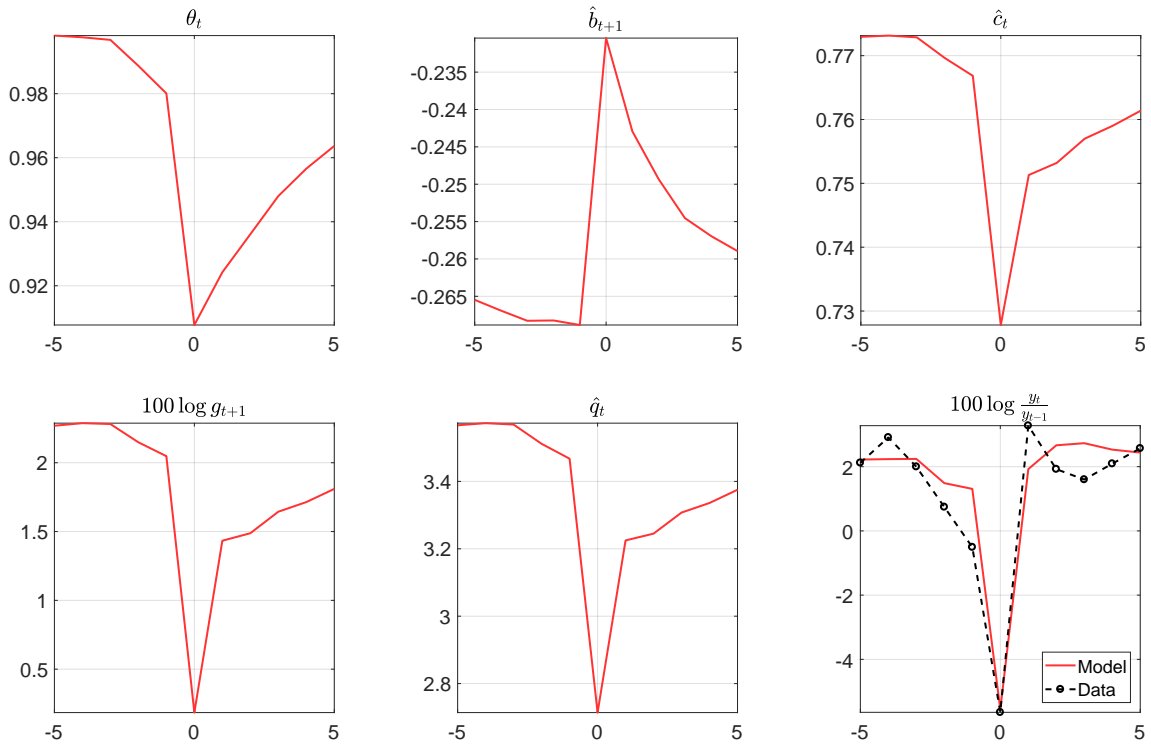

5 Quantitative Results

In this section, I first compare the allocations of private agents and of the macroprudential social planner, and then analyze policy impacts on average growth. I also calculate welfare gains from macroprudential policy and compare these values with the literature. Lastly, I analyze the size of macroprudential taxes. In Appendix E, I conduct a sensitivity analysis with respect to the results.

5.1 Comparing CE and MP Allocations

The difference between the macroprudential social planner and private agents is captured by policy functions. Figure 3 plots consumption $\hat{c}_t^h$, endogenous growth rate g_{t+1}, asset price $\hat{q}_t$, and bond holding $\hat{b}_{t+1}$ for the competitive equilibrium (red solid line) and the

macroprudential social planner (green dashed line) over the bond holding $\hat{b}_t$ when θ_t is 2 standard deviations below its long-run average.[20]

Figure 3: Policy Functions: CE and MP
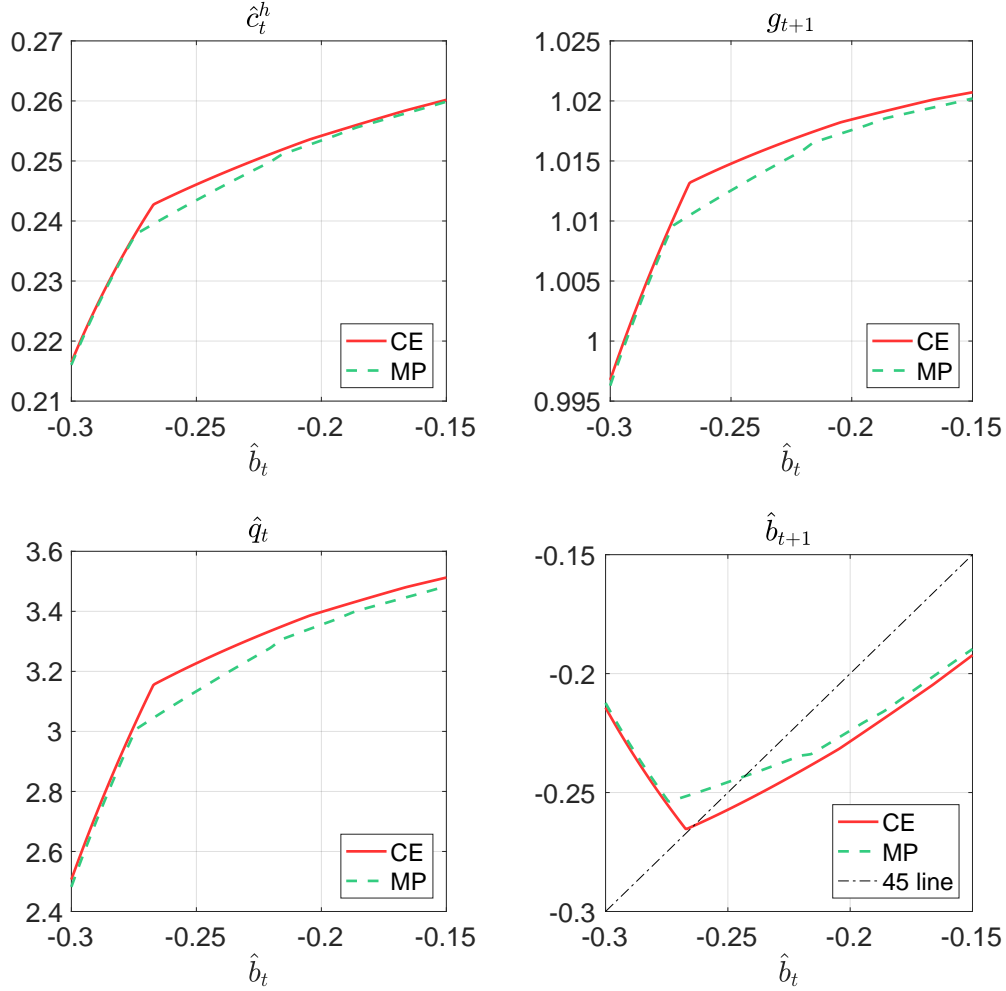
There are kinks in all policy functions due to the presence of the collateral constraint. When the economy starts from a lower bond holding $\hat{b}_t$ (a higher debt to repay), the collateral constraint binds, and private agents have to cut external borrowing and total spending. As a result, both consumption and growth are reduced.

Consistent with the literature, there is an over-borrowing phenomenon in the competitive equilibrium because the social planner chooses a higher bond holding $\hat{b}_{t+1}$ than do private agents. Unlike in the literature, the over-borrowing also has an implication for the endogenous growth rate: The social planner chooses a lower g_{t+1} when the constraint is slack.

Figure 4 displays the ergodic distributions of bond holding $\hat{b}_{t+1}$ and endogenous growth rate g_{t+1}. Compared with private agents, the macroprudential social planner borrows less and thus chooses more mass in the range of higher bond holdings. In terms of the ergodic distribution for g_{t+1}, the social planner has less mass at both extremely low and normal (around 2 percent) growth levels. One can see that the dispersion of growth for MP has been marginally reduced. However, it is unclear whether average growth has been increased or decreased.

Figure 4: Ergodic Distributions: CE and MP
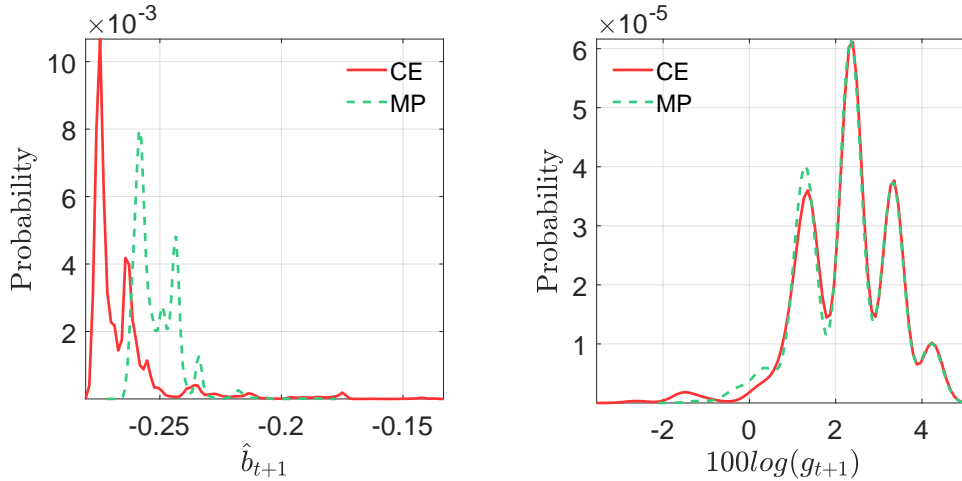
To see the impact of macroprudential policy on average growth and the probability of crisis, Table 3 reports model moments for the social planner and private agents. With macroprudential policy, external borrowing is reduced from 27.18 percent to 25.78 percent, which lowers average growth from 2.315 percent to 2.307 percent. However, the policy also reduces the probability of crisis from 6.23 percent to 1.89 percent. Hence, the economy becomes more resilient.

Figure 5 reports the event window as before but also plots the dynamics of variables for the social planner given the same exogenous shock θ_t. One can see that the probability of crisis has been reduced by the social planner in the last panel of Figure 5. Furthermore,

Table 3: Moments: CE and MP

Moments	CE	MP
Average GDP growth (%)	2.315	2.307
Probability of crisis (%)	6.23	1.89
NFA-GDP ratio (%)	-27.18	-25.78
Consumption-GDP ratio (%)	77.53	77.65
Correlation between current account and output	-0.22	-0.37

the planner chooses a higher bond holding in normal periods and thus suffers less when the really big shock hits at time 0. As a result, the social planner cuts consumption and growth-enhancing expenditures less during crises.

Figure 5: Event Window: CE and MP
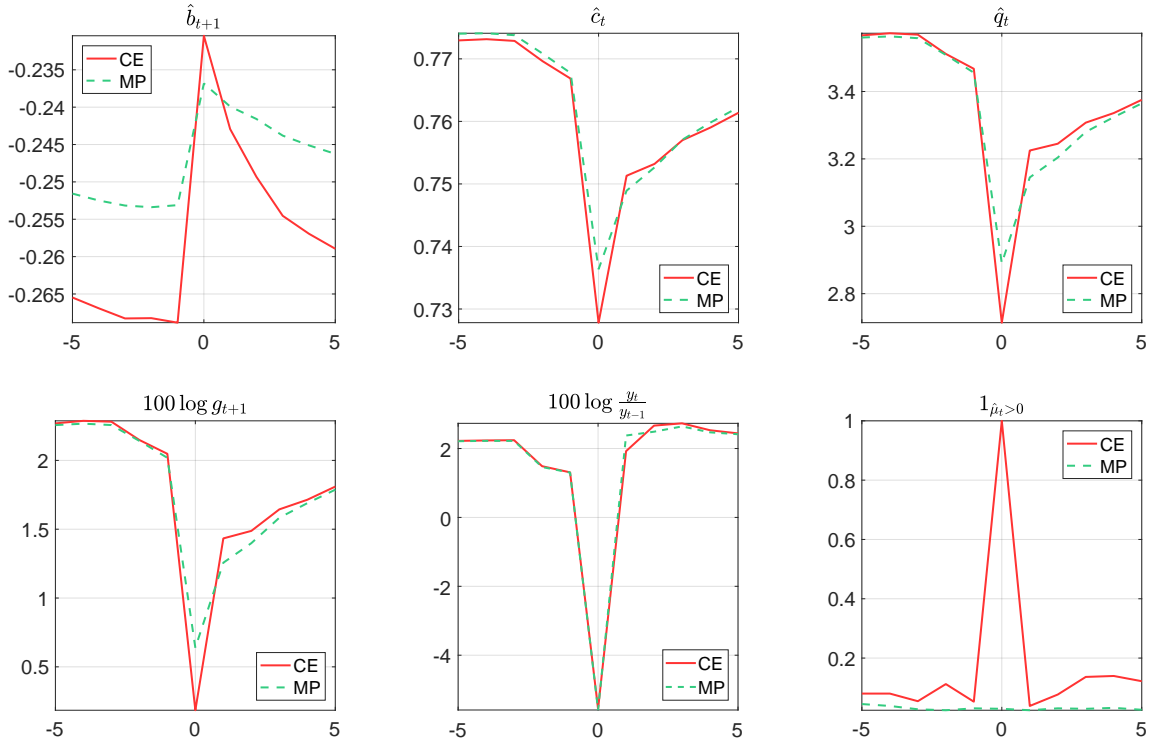
However, macroprudential policy also reduces borrowing and thus the endogenous growth in normal periods. To show its impact, Figure 6 plots the transition dynamics from competitive equilibrium to the equilibrium chosen by the social planner.[21] On balance,

---

[21]The transition dynamics is constructed by first running 1,000 simulations of 1,020 periods for com-

the macroprudential social planner borrows less than private agents, which reduces both consumption and endogenous growth. However, the economy becomes more resilient and has a lower probability of crisis. Therefore, consumption converges to a higher level. But the endogenous growth rate g_{t+1} only converges to a lower level because the economy borrows less in the long run.

Figure 6: Transition Dynamics: CE and MP
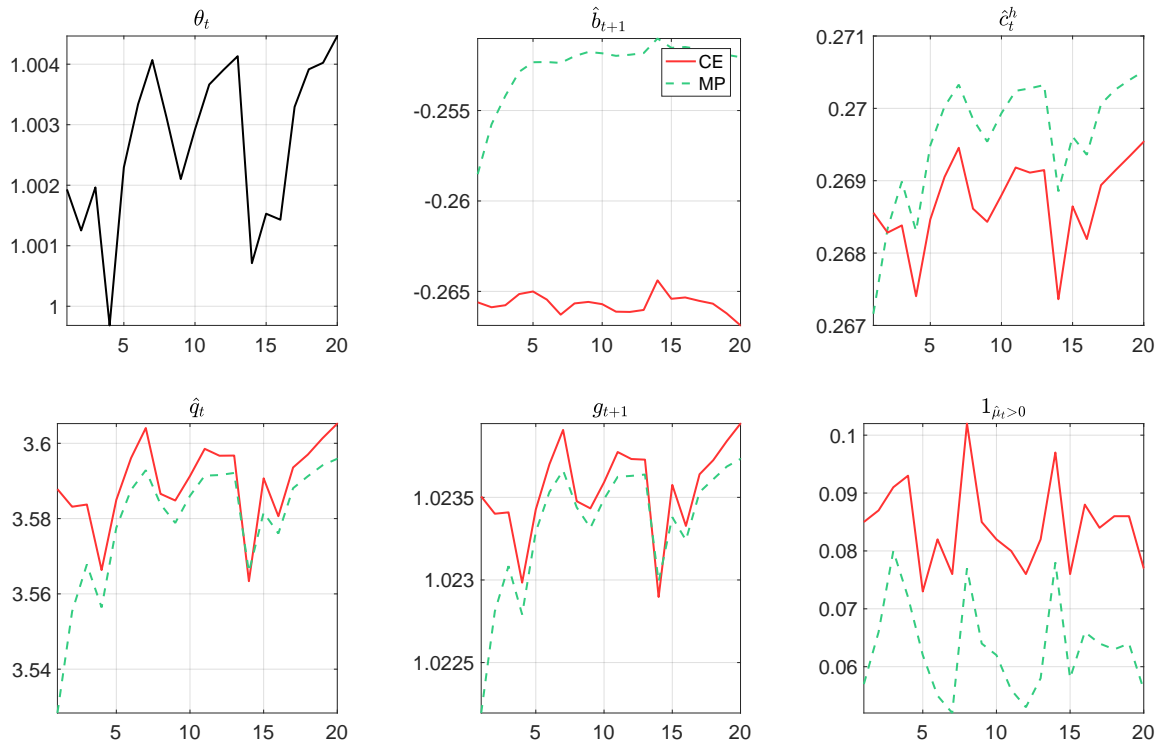
5.2 Policy Impacts on Average Growth

This model allows for an analysis of policy impacts on average growth. It is clear that macroprudential policy increases the endogenous growth rate g_{t+1} during crises but reduces it in normal periods. Even though the policy lowers the volatility of growth unambiguously, its impacts on average growth are ambiguous in theory.

In the baseline calibration, there is a negative relationship between average growth and financial stability for macroprudential policy. But a more general question is which

petitive equilibrium and then introducing the social planner from period 1,001.

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parameters govern this relationship. To answer this question, I proceed by simplifying the model such that it can almost completely be solved analytically.

Instead of using the existing log AR(1) process for θ_t, I assume that $\theta_t = 1$ for all t, and that it falls to 0.9 in the second period, with a probability $p \in [0, 1]$. Furthermore, the economy is unconstrained in a steady state, and I need to change β such that $\beta(1+r)g_{ss}^{-\gamma} = 1$, where $g_{ss} = 1.023$, as in the baseline calibration. I keep other parameter values the same as before. Hence, crisis occurs in the economy when $\theta_2 = 0.9$ and the collateral constraint binds.

I plot the average growth chosen by the private agents and by the social planner in Figure 7.[22] Whether the social planner increases or decreases average growth depends on two parameters: The probability of negative shock p and the tightness of the collateral constraint ϕ. Intuitively, the macroprudential social planner can increase average growth because she reduces the cost of crisis and thus raises the growth rate during a crisis. However, a crisis occurs with probability p, and its cost depends on the tightness of the collateral constraint. When p is higher or ϕ is lower, macroprudential policy is very beneficial, since the expected cost of crisis is relatively large. Hence, the policy can increase average growth in these scenarios.

Figure 7: Policy Impacts on Average Growth: CE and MP

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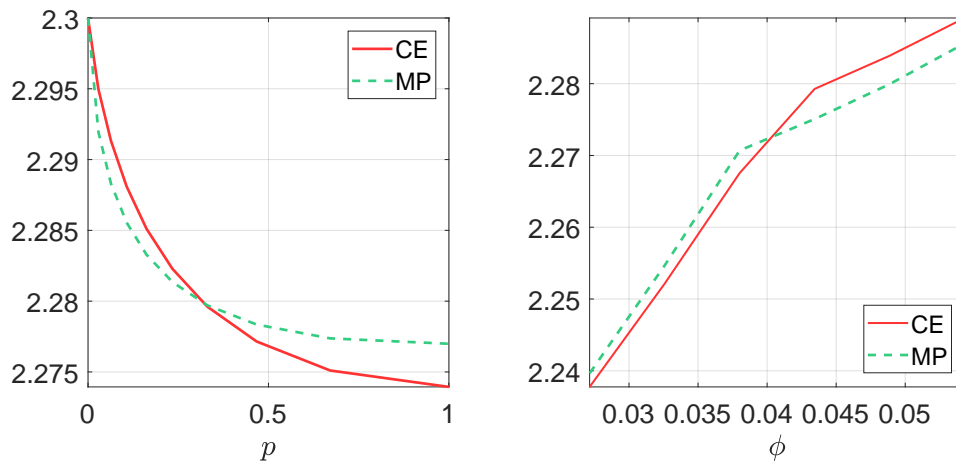
[22] I run 100-period simulations in two separate states to calculate average growth: $\theta_2 = 0.9$ in state L and $\theta_2 = 1$ in state H. The growth rate for each simulation is calculated as follows:

$$G^i = \left(\Pi_{t=1}^{100} g_{t+1}\right)^{\frac{1}{100}}, \text{ where } i \in \{H, L\}$$

Therefore, average growth is $p * G^L + (1 - p) * G^H$.

I also find that the magnitude of the impacts is small (see Figure 7 and Table 3). This is because there is an optimal rate of growth defined by the technology $\Psi(z_{t+1}, z_t)$. Macroprudential policy does not change this function directly but only changes the marginal valuation of wealth. Furthermore, any changes in the growth rate have non-trivial effects on welfare (see Lucas (1987) and Barlevy (2004)). Hence, if the optimal policy has to affect growth negatively in order to increase financial stability, a planner will tend to choose a policy that changes growth only by a small amount. Otherwise, it is too costly for social welfare.

5.3   Welfare Gains

To calculate the welfare gains from macroprudential policy, I define a variable $\Delta^{MP}(\hat{b}_t, \theta_t)$, which compares two utilities and converts their difference into consumption equivalents:

$$\Delta^{MP}(\hat{b}_t, \theta_t) = 100 \left[\left(\frac{\hat{V}^{MP}(\hat{b}_t, \theta_t)}{\hat{V}^{CE}(\hat{b}_t, \theta_t)} \right)^{\frac{1}{1-\gamma}} - 1 \right] \tag{19}$$

where $\hat{V}^i(\hat{b}_t, \theta_t)$ is a normalized value function and $i \in \{CE, MP\}$.

$\Delta^{MP}(\hat{b}_t, \theta_t)$ depends on state variables $\{\hat{b}_t, \theta_t\}$, and I plot it in Figure 8.[23] Consistent with the literature, it peaks in the region where the magnitude of externalities is at its maximum. It becomes smaller when the economy has a higher amount of bond holding, since the probability of future crisis is lower. It also becomes smaller when the economy has a lower amount of bond holding, i.e. when the constraint binds. The macroprudential social planner chooses the same allocation as the private agents in these regions. Hence, the welfare gains are small.

To understand the average benefit of macroprudential policy, I also define a variable EV^{MP} as follows:

$$EV^{MP} = E\left[\Delta^{MP}(\hat{b}_t, \theta_t) \right] \tag{20}$$

where the expectation is taken using the ergodic distribution of $\hat{b}_t$ and θ_t in competitive equilibrium.
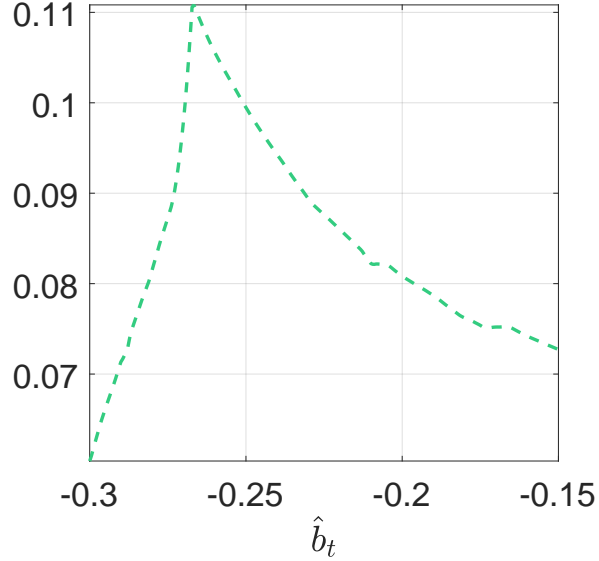
The unconditional welfare gains from the macroprudential social planner EV^{MP} are equivalent to a 0.06 percent permanent increase in annual consumption, the same range

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[23]Like the policy functions, $\Delta^{MP}(\hat{b}_t, \theta_t)$ is plotted over the bond space $\hat{b}_t$ when the shock θ_t is 2 standard deviations below its long-run average.

Figure 8: Welfare Gains (%): MP

as in the literature. Hence, endogenous growth does not fundamentally change the benefit of macroprudential policy.

To understand the reason, I decompose the overall welfare gains into two channels: One is a cyclical component of consumption $\hat{c}_t^h$, a traditional channel as in the literature, and the other is a trend component of consumption, i.e., productivity z_t, a new channel with endogenous growth. Specifically, utilities depend on the net consumption series $\{c_t^h\}_{t=0}^\infty$, which in turn is the product of the cyclical component of consumption $\{\hat{c}_t^h\}_{t=0}^\infty$ and the trend component of consumption $\{z_t\}_{t=0}^\infty$. The difference between endogenous and exogenous growth is whether policies can affect the trend component of consumption. If I find that gains come through the cyclical rather than the trend component of consumption, it is not surprising that endogenous growth does not fundamentally change the benefit of optimal policy.

To accomplish that, I run 1,000 simulations and get both cyclical and trend components of consumption for the competitive equilibrium and the social planner. To control for the trend (cyclical) component of the consumption channel, I multiply the trend (cyclical) component of consumption in competitive equilibrium by the cyclical (trend) component of consumption under the social planner to construct a counter-factual consumption. I then compare the utility of this counter-factual consumption with the utility of consump-

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tion in competitive equilibrium. The difference between these two is considered as gains through the cyclical (trend) component of consumption channel.

Table 4 reports the results. Indeed, gains through the cyclical component of consumption channel are reinforced by endogenous growth: a 0.40 percent permanent increase in annual consumption, which is much larger than those found in the literature. However, there are welfare losses through the trend component of the consumption channel, since the policy reduces average growth. Even if the magnitude of reduction is small, 0.01 percentage point, the cost in terms of welfare is large, a 0.34 percent permanent decrease in annual consumption. Overall, macroprudential policy is still desirable, but the gains are no larger than those in the models with exogenous growth.

Table 4: Source of Welfare Gains (%)

|	Overall	Trend Consumption Channel	Cyclical Consumption Channel
MP	0.06	-0.34	0.40

5.4 Policy Instruments

Figure 9 shows the macroprudential tax on capital flows $\tau_t^{MP,b}$.[24] The tax rate varies from 0 to 5 percent, depending on the state variable $\hat{b}_t$, and I find that it is 1.28 percent on average. As explained before, the macroprudential social planner cannot change the allocation when the constraint binds, and I set the tax rate at zero in these regions. Consistent with the literature, the tax rate peaks in the region where the magnitude of externalities is at its maximum. The tax approaches zero when the economy has sufficient bond holdings $\hat{b}_t$.

6 Extension: Other Policy Instruments

In this section, I introduce a social planner who has two instruments. For the sake of comparison, I call her a *multi-instrument social planner* (MI). Unlike the macroprudential social planner, who only has one instrument to influence the level of spending, the multi-instrument social planner can also change the composition of spending. Hence, an additional policy is needed to implement her allocation. As I will explain later, this new policy can be interpreted as a stimulus policy.

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[24]As before, I plot it over the bond holding $\hat{b}_t$ when the shock θ_t is 2 standard deviations below its long-run average.

Figure 9: Macroprudential Tax on Capital Flows
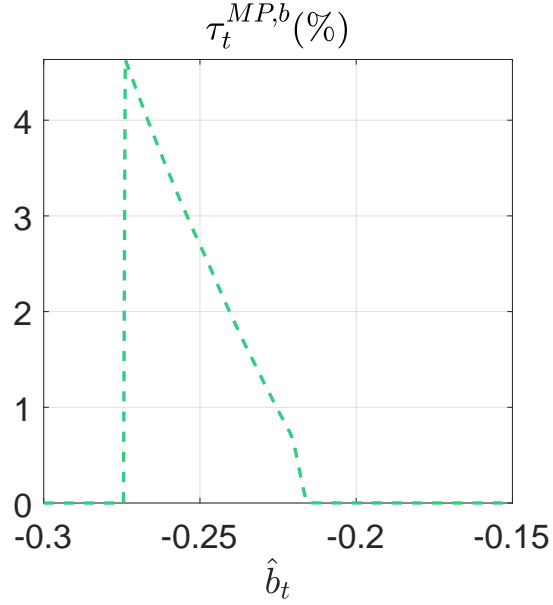
Like the macroprudential social planner, the multi-instrument social planner chooses allocation on behalf of private agents subject to the resource constraint (8) and the collateral constraint (5). Differently, she only has the asset equation (11) as an implementation constraint, not the growth equation (10). Therefore, she can choose z_{t+1} without restrictions. Specifically, her maximization problem can be written as

$$
V_t^{MI}(z_t, b_t, \theta_t) = \max_{c_t^h, z_{t+1}, b_{t+1}, q_t} u\left(c_t^h\right) + \beta E\left[V_{t+1}^{MI}(z_{t+1}, b_{t+1}, \theta_{t+1})\right]
$$

$$
\text{s.t.} \quad c_t^h + h z_t + \Psi(z_{t+1}, z_t) + b_{t+1} = \theta_t z_t + (1+r) b_t,
$$

$$
-b_{t+1} \le \phi q_t,
$$

$$
u'(c_t^h) q_t = \underbrace{\beta E_t\left[u'(c_{t+1}^h)\left(\alpha \theta_{t+1} z_{t+1} + q_{t+1}\right)\right]}_{G(z_{t+1}, b_{t+1})}.
$$

where the last constraint is the Euler equation of choosing a productive asset.

The maximization problem implies the following optimality conditions for each period:

$$
\begin{aligned}
\lambda_t^{MI} &= u'(c_t^h) - \xi_t^{MI} u''(c_t^h) q_t \\
\lambda_t^{MI} \Psi_{1,t} &= \xi_t^{MI} G_{1,t} + \beta E_t \left[\lambda_{t+1}^{MI} (\theta_{t+1} - h - \Psi_{2,t+1}) \right] \quad (21) \\
\phi \mu_t^{MI} &= \xi_t^{MI} u'(c_t^h) \\
\lambda_t^{MI} &= \mu_t^{MI} + \xi_t^{MI} G_{2,t} + \beta(1+r) E_t \left[\lambda_{t+1}^{MI} \right]. \quad (22)
\end{aligned}
$$

where λ_t^{MI}, μ_t^{MI}, and ξ_t^{MI} are Lagrangian multipliers associated with the budget constraint, collateral constraint, and implementation constraint, respectively.

Wedge in Marginal Valuation of Wealth: The main difference between CE and MI is reflected in the marginal valuation of wealth, λ_t^{CE} and λ_t^{MI}. Like the macroprudential social planner, the multi-instrument social planner values wealth more than private agents do, due to the term $-\xi_t^{MI} u''(c_t^h) q_t$, capturing pecuniary externalities in the economy. Unlike the macroprudential social planner, she can choose productivity freely, as in equation (21), and does not have an additional term in the wedge, as in $\lambda_t^{MP} - \lambda_t^{CE}$.

The wedge in the marginal valuation of wealth also has an implication for external borrowing and growth. Unlike the macroprudential social planner, who is constrained to implement the same allocation as private agents when the constraint binds, the multi-instrument social planner shifts spending from growth-enhancing expenditures to consumption. By doing so, she can increase the asset price and thus relax the collateral constraint. When the collateral constraint is slack, she borrows less, for the same reason as the macroprudential social planner does. However, she also chooses a higher growth rate than do private agents so as to offset the negative effect of decreased borrowing on growth.

Implementation: I assume that the social planner has access to a tax $\tau_t^{MI,b}$ on capital flows, a subsidy $\tau_t^{MI,z}$ on growth-enhancing expenditures, and a lump-sum transfer T_t^{MI}. The budget constraint of private agents becomes

$$
c_t^h + h z_t + \left(1 - \tau_t^{MI,z}\right) \Psi(z_{t+1}, z_t) + q_t n_{t+1} + \left(1 - \tau_t^{MI,b}\right) b_{t+1} = y_t + q_t n_t + (1+r) b_t + T_t^{MI}
$$

where $T_t^{MI} = -\tau_t^{MI,z} \Psi(z_{t+1}, z_t) - \tau_t^{MI,b} b_{t+1}$.

Proposition 2. *Decentralization with Two Instruments*
The multi-instrument social planner's allocation can be implemented by a tax $\tau_t^{MI,b}$ on capital flows and a subsidy $\tau_t^{MI,z}$ on growth-enhancing expenditures, which are rebated to

private agents with a lump-sum transfer T_t^{MI}. Furthermore, $\tau_t^{MI,b}$ and $\tau_t^{MI,z}$ are given by

$$\tau_t^{MI,z} = \frac{\beta g_{t+1}^{-\gamma} E_t \left[\hat{c}_{t+1}^{-\gamma} \tau_{t+1}^{MI,z} \Psi_{2,t+1} + \gamma \phi \hat{\mu}_{t+1}^{MI} \hat{q}_{t+1} \left(\hat{c}_{t+1}^h \right)^{-1} \left(\theta_{t+1} - h - \Psi_{2,t+1} \right) \right]}{\Psi_{1,t} \left(\hat{c}_t^h \right)^{-\gamma}}$$

$$- \frac{\gamma \phi \hat{q}_t \left(\hat{c}_t^h \right)^{-1} \hat{\mu}_t^{MI} \Psi_{1,t} - \phi \hat{\mu}_t^{MI} \left(\hat{c}_t^h \right)^{\gamma} g_{t+1}^{-\gamma} \hat{G}_{1,t}}{\Psi_{1,t} \left(\hat{c}_t^h \right)^{-\gamma}},$$

$$\tau_t^{MI,b} = - \frac{\gamma \phi \hat{q}_t \left(\hat{c}_t^h \right)^{-1} \hat{\mu}_t^{MI} - \phi \hat{\mu}_t^{MI} \left(\hat{c}_t^h \right)^{\gamma} g_{t+1}^{-\gamma} \hat{G}_{2,t} - \beta g_{t+1}^{-\gamma} \left(1 + r \right) E_t \left[\gamma \phi \hat{q}_{t+1} \left(\hat{c}_{t+1}^h \right)^{-1} \hat{\mu}_{t+1}^{MI} \right]}{\left(\hat{c}_t^h \right)^{-\gamma}}$$

Proof. See Appendix D.2.

I need two instruments $\left\{ \tau_t^{MI,z}, \tau_t^{MI,b} \right\}$ to close the wedge between λ_t^{MI} and λ_t^{CE} on allocations, since it affects two decision margins in the economy. Both instruments are used ex-ante and ex-post. The only instrument I call *macroprudential policy* is the ex-ante capital flow tax $\tau_t^{MI,b}$, the instrument that is available to the macroprudential social planner in the benchmark analysis. When the tax is used ex-post, I call it a *stimulus policy*, a category which also includes the subsidy $\tau_t^{MI,z}$ on growth-enhancing expenditures. Hence, the stimulus policy can be used for both ex-ante and ex-post intervention. The reason that the ex-ante growth subsidy $\tau_t^{MI,z}$ also belongs to the stimulus policy is that the multi-instrument social planner uses it to offset the negative effect of ex-ante capital flows tax $\tau_t^{MI,b}$ on growth.

Discussion on the Two Social Planners: The main difference between the two social planners is the availability of instruments, which is related to the on-going policy debate on ex-ante versus ex-post policy intervention. The macroprudential social planner only intervenes ex-ante, while the multi-instrument social planner intervenes both ex-ante and ex-post. I choose the macroprudential social planner as the benchmark analysis to stay in line with the literature and to focus on the differences between exogenous and endogenous growth.

Furthermore, macroprudential policy is more realistic and relevant for an emerging market to use to smooth boom-bust cycles in capital flows. Empirical results on the effectiveness of macroprudential policy are mostly supportive. For example, Lim et al. (2011) and Bruno et al. (2017) have estimated the effectiveness of macroprudential tools using comprehensive data and argue that such tools are effective in reducing the pro-

cyclicality of shocks (see Galati and Moessner (2017) for a summary).[25]

The stimulus policy, however, is hard to implement. In my model, it includes an ex-ante growth subsidy and an ex-post policy intervention. For the ex-post intervention, both capital flow taxes and growth subsidies are used to change the composition of spending in order to raise asset prices and relax the borrowing constraint. Such intervention is required during crises and potentially incurs some cost (see Jeanne and Korinek (2013) and Benigno et al. (2016)). The ex-ante growth subsidy is used to correct pecuniary externalities rather than externalities in endogenous growth, as in the literature. But there exists a fundamental implementation issue because one needs to identify the source of economic growth—i.e., $\Psi(z_t, z_{t+1})$—in order to impose the subsidy. The identification failure typically leads to the futility of a subsidy policy. Indeed, there is little evidence for positive effects of subsidies on productivity (see Westmore (2013)).

## 6.1 Comparing CE, MP, and MI Allocations

Figure 10 compares policy functions of CE, MP, and MI. Unlike private agents and the macroprudential social planner, the multi-instrument social planner can shift resources from growth-enhancing expenditures to consumption when the collateral constraint binds. This behavior comes at a second-order cost, since it distorts the first-order conditions of private agents in choosing bond holdings and productivity. However, there is a first-order gain, because it increases the asset price $\hat{q}_t$ and thus relaxes the collateral constraint. As a result, the social planner can borrow *more* even during a crisis, and the crisis is not as costly as in competitive equilibrium; one can see that consumption $\hat{c}_t^h$, endogenous growth g_{t+1}, and asset price $\hat{q}_t$ are much higher. The multi-instrument social planner's allocation in crisis also has implications for her allocation in normal periods: She actually chooses *fewer* bond holdings and a *higher* endogenous growth rate than do private agents, and the constraint becomes binding with a higher level of bond. Hence, the economy ends up with more financial instability.

Figure 11 plots the ergodic distributions of bond holdings and endogenous growth rate. Unlike the macroprudential social planner, the multi-instrument social planner chooses more mass in the range of lower bond holdings $\hat{b}_{t+1}$. Like the macroprudential social planner, she also has less mass at both extremely low and normal (around 2 percent)

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[25]There are exceptions. For example, Fernández et al. (2015) cast some doubts on the effectiveness of macroprudential policies, since they find the instruments are acyclical, which counters the theoretical predictions for prudential tools. Policies' effectiveness depends crucially on their design. There are issues that might affect their effectiveness. For example, Bengui and Bianchi (2014) investigate the issue of leakage, and Dogra (2014) investigates the issue of private information.

Figure 10: Policy Functions: CE, MP and MI
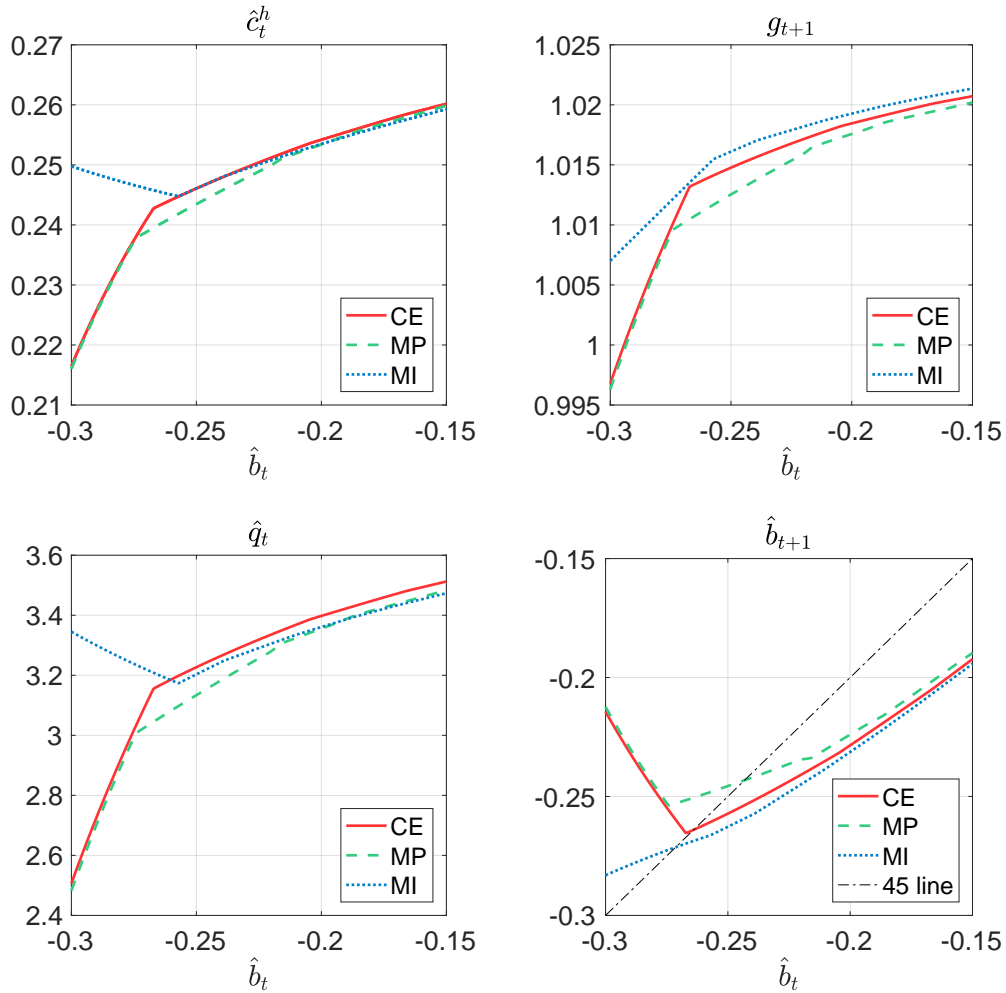
growth levels than at the competitive equilibrium. As a result, the dispersion of growth has been reduced, but the effect on average growth is unclear.

Table 5 reports model moments of CE, MP, and MI. From the economy in CE to MI, the probability of crisis has increased from 6.2 percent to 14.2 percent, and average growth is reduced by 0.03 percentage point. This is counter-intuitive but reasonable because the crisis is less costly with the stimulus policy and the social planner strikes a new balance between impatience and precautionary motive. Given that the private agent is impatient, the social planner finds it optimal to borrow *more* and hit the collateral constraint more frequently, since she can intervene ex-post to reduce the cost of crisis. But the ex-post intervention requires a shift of spending from growth-enhancing expenditures

Figure 11: Ergodic Distributions: CE, MP and MI
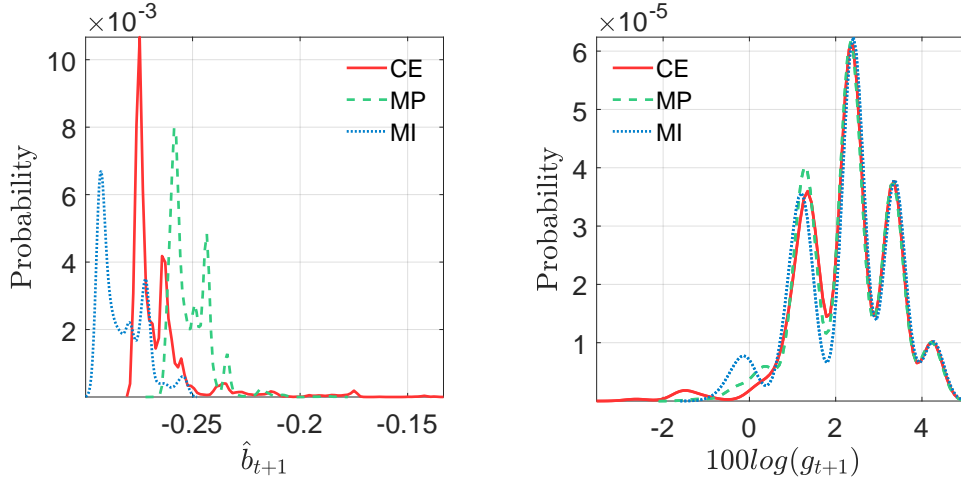
to consumption. As a result, average growth rate is even lower in MI than in MP.

Table 5: Moments: CE, MP, and MI

| Moments	CE	MP	MI
Average GDP growth (%)	2.315	2.307	2.289
Probability of crisis (%)	6.23	1.89	14.23
NFA-GDP ratio (%)	-27.18	-25.78	-28.98
Consumption-GDP ratio (%)	77.53	77.65	77.58
Correlation between current account and output	-0.22	-0.37	-0.54

Even if the multi-instrument social planner lowers average growth, she can still smooth the economy. Figure 12 reports the event window as before. One can see that consumption and asset prices fall *less* in MI than in CE and MP during crises. Furthermore, the endogenous growth rate g_{t+1} during crises falls *less* in MI than in CE but *more* than in MP because the social planner shifts resources from growth-enhancing expenditures to consumption. Due to the existence of ex-post intervention, the social planner borrows *more* ex-ante and hits the borrowing constraint more frequently.

Unlike the macroprudential social planner, who increases endogenous growth g_{t+1} during the crisis but reduces it in normal periods, the multi-instrument planner actually increases endogenous growth g_{t+1} in the short run but reduces it in the long run. To demonstrate the difference, I show the transition dynamics in Figure 13.

The multi-instrument social planner generates a short-run boom in consumption and growth, since she can intervene ex-post and thus borrows more ex-ante. However, growth

Figure 12: Event Window: CE, MP, and MI
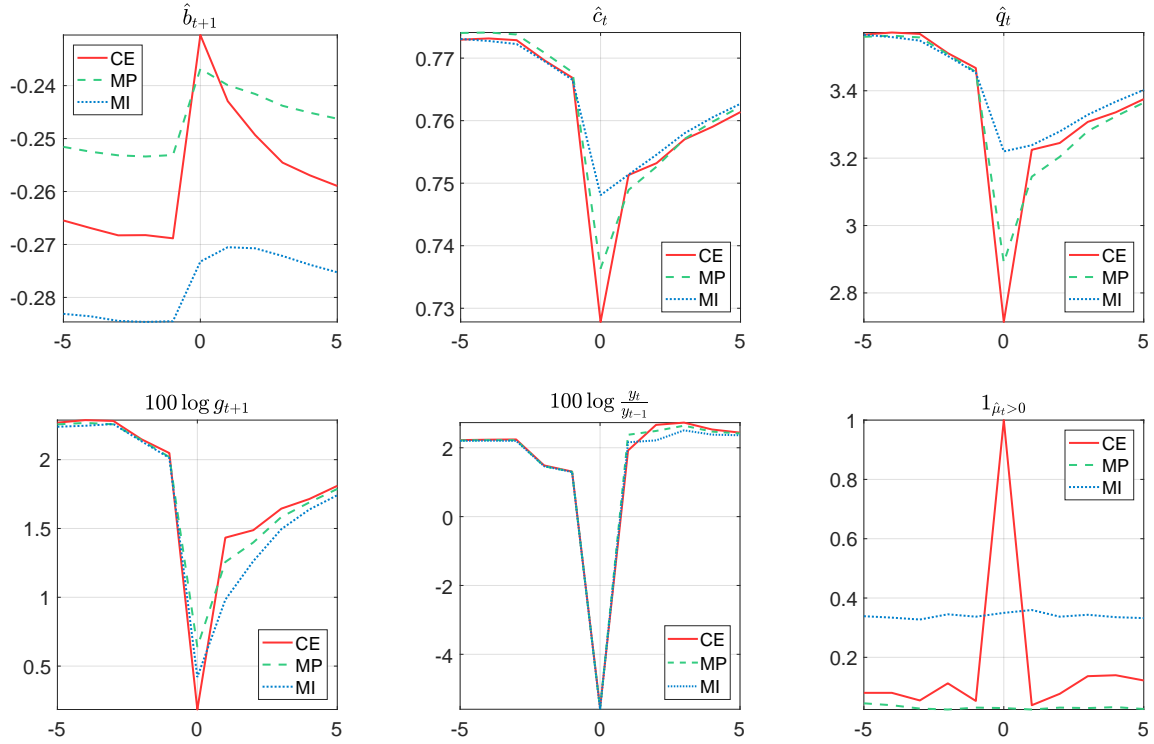
converges to a lower level in the long run because resources are used to serve a higher level of external debt. Therefore, the multi-instrument social planner actually faces a trade-off between short and long-run growth. Furthermore, I find that the short-run boom in average growth lasts for 18 years.

6.2 Policy Impacts on Average Growth: MI and MP

Given that the multi-instrument social planner has access to the stimulus policy, one natural question is whether she can increase average growth. To answer this question, Figure 14 shows average growth in the simplified version of the model, as before. One can see that the social planner indeed increases average growth all the time. The stimulus policy allows the social planner to intervene ex-post, which mitigates the cost of crises. Furthermore, the stimulus policy also offsets the negative effect of macroprudential policy on growth in normal periods. Therefore, average growth is increased for the multi-instrument

Figure 13: Transition Dynamics: CE, MP, and MI
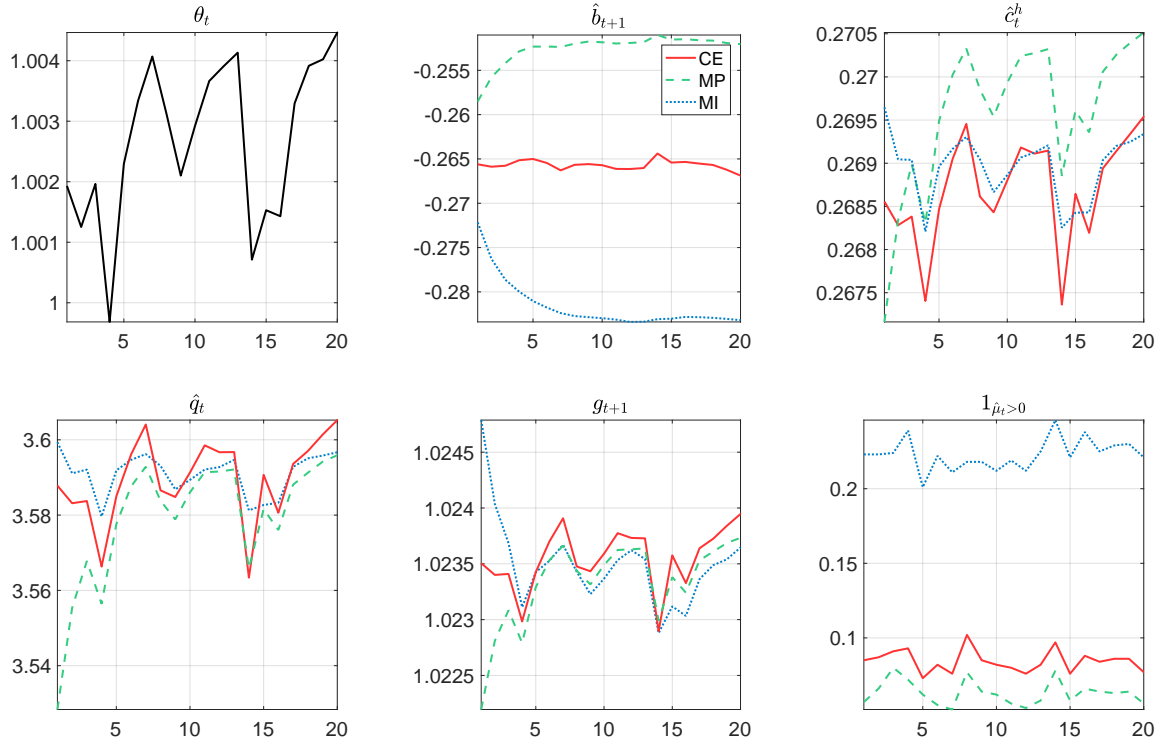
social planner.[26]

6.3 Welfare Gains: MI and MP

Unsurprisingly, the multi-instrument social planner generates larger welfare gains than does the macroprudential social planner thanks to the availability of stimulus policy. Figure 15 plots conditional welfare gains. The gains become larger when the constraint binds tighter (a lower bond $\hat{b}_t$), reflecting the importance of stimulus policy.

Average welfare gains are equivalent to a 0.24 percent permanent increase in annual consumption, and the source of the gains is the cyclical component of the consumption channel, as before (see Table 6). Furthermore, the gains from this channel do not in-

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[26]The results are different from our baseline calibration, where average growth for the multi-instrument social planner is decreased. As I explained before, given that the cost of financial crisis is reduced, the social planner finds it optimal to hit the borrowing constraint more frequently. Average growth rate is thus reduced because the resources are shifted from growth-enhancing expenditures towards consumption. However, this channel is not in the simplified version of the model since the probability of crisis is given by the exogenous parameter p.

Figure 14: Policy Impacts on Average Growth: CE, MP, and MI
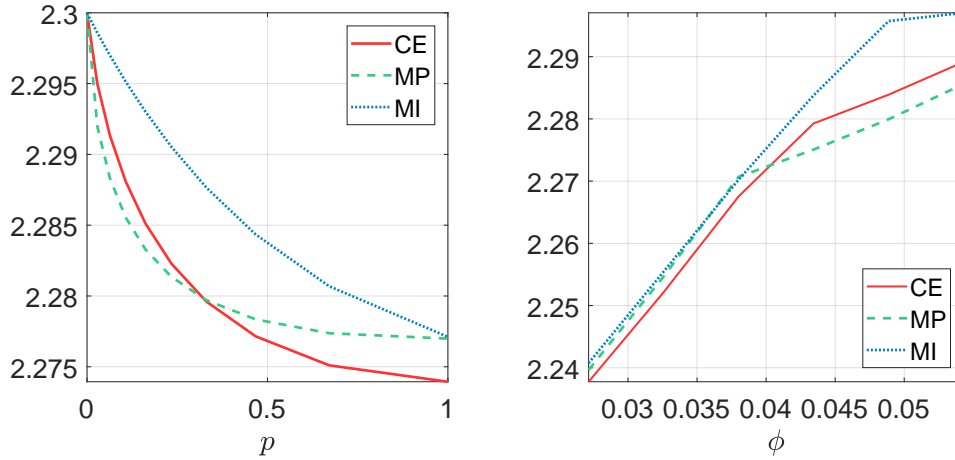
crease with two instruments. Instead, the welfare loss in the trend component of the consumption channel is significantly reduced, from a 34 percent to a 13 percent permanent decrease in annual consumption. Hence, the stimulus policy reduces the negative impact of macroprudential policy on growth and thus on welfare.

Table 6: Source of Welfare Gains (%): MP and MI

	Overall	Trend Consumption Channel	Cyclical Consumption Channel
MP	0.06	−0.34	0.40
MI	0.24	−0.13	0.38

6.4 Policy Instruments

Figure 16 plots $\left\{\tau_t^{MI,b}, \tau_t^{MI,z}\right\}$ over the bond space $\hat{b}_t$ when θ_t is 2 standard deviations below its long-run average. For the capital flow tax $\tau_t^{MI,b}$, one can see that it is positive when the constraint is slack (used ex-ante), just as it is for the macroprudential social planner. However, when the initial wealth is low (i.e., $\hat{b}_t$ is low and the constraint binds), the tax becomes negative, meaning that the social planner wants to encourage borrowing. This is because she can relax the constraint in the bad state and thus borrow more than private agents. The growth subsidy $\tau_t^{MI,z}$ is positive in normal periods, since the social planner wants the stimulus policy to offset the negative effect of macroprudential policy on growth. When the constraint binds, it is negative, since the social planner wants to

Figure 15: Welfare Gains (%): MI
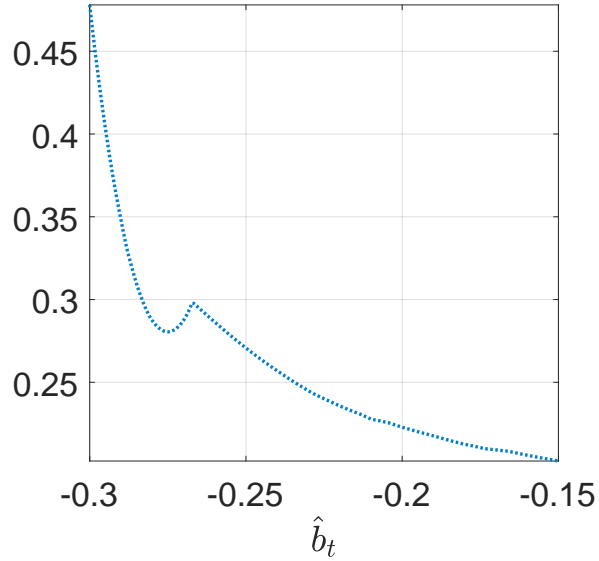
shift resources from growth-enhancing expenditures to consumption so as to relax the borrowing constraint.

Table 7 reports the average of capital flows tax and growth subsidy. I find that, on average, capital flow tax is 1.12 percent. The ex-ante tax-macroprudential policy—is 1.1 percent, and the ex-post tax is 1.19 percent. The growth subsidy, on average, is 1.00 percent. The ex-ante subsidy is 1.87 percent, and the ex-post subsidy is -1.78 percent. Based on these results, one might argue that the existence of ex-post intervention reduces the magnitude of ex-ante intervention, as in Jeanne and Korinek (2013). However, this result depends on calibrations (see the sensitivity analysis in Appendix E).

Table 7: Policy Instruments (%): MP and MI

	Capital Flows Tax	Growth Subsidy
MP	1.28	N.A.
MI	1.12	1.00
MI (Ex-ante)	1.10	1.87
MI (Ex-post)	1.19	-1.78

Figure 16: Two Instruments (%): MI
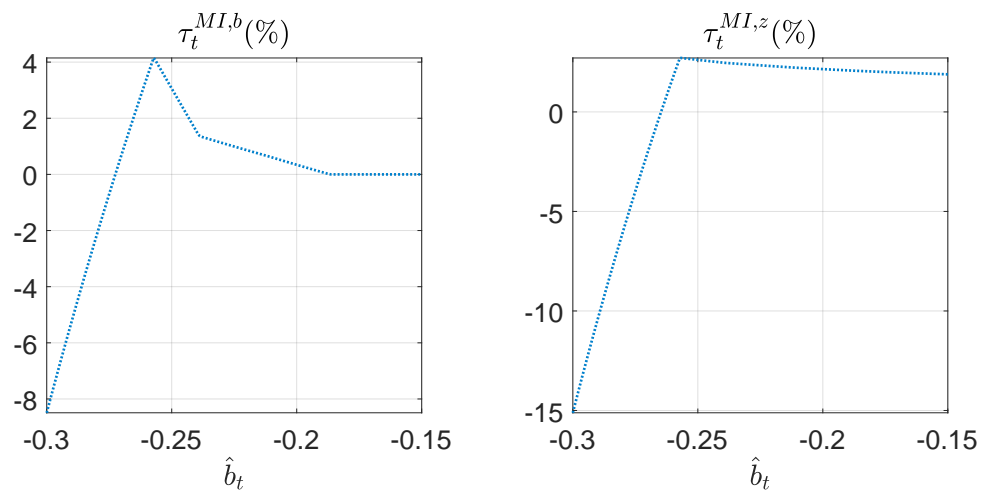
7   Conclusion

This paper introduces endogenous growth into a model with occasionally binding collateral constraints of the type that has been used previously in the literature on macroprudential policy. In the previous literature, binding constraints did not have a long-run impact on output. By contrast, in my model, they do, which increases their cost and presumably might reinforce the case for macroprudential policy. My model thus lends itself to analyzing the role of macroprudential policy in the context of a tradeoff between growth and financial stability.

The impact of macroprudential policy on average growth is, in general, ambiguous. Macroprudential policy reduces the frequency of crises and their impact on growth but comes at the cost of reducing borrowing and growth in good times. To resolve this ambiguity, I look at a calibrated version of the model.

In the quantitative analysis, I find that optimal macroprudential policy substantially reduces the frequency of crisis but has a very small negative effect on average growth. As is known in the literature, changes in average growth have very large welfare impacts (see Lucas (1987) and Barlevy (2004)). Given that optimal macroprudential policy has to lower average growth in order to increase financial stability, it does not change growth by a large amount, because even a small reduction in growth is costly in terms of welfare. Quantitatively, a 0.01 percentage point reduction in average growth leads to a welfare loss equivalent to a 0.34 percent permanent decrease in annual consumption.

Nevertheless, macroprudential policy is still desirable because it reduces the probability of crisis and smooths consumption. The benefits from consumption smoothing actually outweigh the welfare loss from the reduction in average growth. Overall, welfare gains are at the magnitude of a 0.06 percent permanent increase in annual consumption, which is in the same range as in the existing literature.

The model with endogenous growth also allows me to analyze the role of a stimulus policy that is used both ex-ante and ex-post. When such a policy is available, much larger welfare gains can be generated, since the cost of crises is reduced by the ex-post intervention. Ex-ante, macroprudential policy is used to correct over-borrowing in the credit market, and the stimulus policy is used to offset the negative impact of macroprudential policy on growth. Optimal policy thus leads to a short-run boom in growth and consumption, which significantly reduces the welfare loss from a reduction in average growth. In the long run, growth converges to a lower level, since resources are used to serve a higher level of external borrowing. However, the short-run boom in average growth lasts for 18 years.

This paper is suitable fodder for policymakers' reflections about their policies' impacts on average growth and financial stability. One general message is that macroprudential policy only marginally lowers average growth to enhance financial stability. Therefore, it is still desirable to use macroprudential policy, even taking into account its negative impact on average growth. Furthermore, it is always desirable to have a stimulus policy in addition to macroprudential policy, since these two policies complement each other in mitigating the cost of financial crises.

To the best of my knowledge, this is the first paper to analyze the impact of macroprudential policy on growth. Hence, there are many unsolved, interesting questions that I leave for future research. First, my paper is about the role of macroprudential policy in capital flows. However, many countries, including advanced economies, adopted macroprudential policies on other financial markets after the 2008-09 Global Financial Crisis. It would be interesting to look at the effects of other macroprudential policies (leverage ratio, capital requirement, etc). Second, my paper abstracts from the risk-taking behavior in the economy. In the model, macroprudential policy negatively affects growth because it restricts the amount of funding to productive projects. However, private agents might respond to the policy by taking on riskier projects. Such risk-taking behavior might be socially inefficient, even if it is privately optimal. In the end, excessive risk-taking behavior might lower average growth. Therefore, it may be interesting to see whether average growth is further driven down by optimal policy.

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A   Data Source

The sample includes the following 55 countries:

Algeria	Argentina	Australia	Austria	Belgium
Brazil	Canada	Chile	China	Colombia
Cote d'Ivoire	Croatia	Czech Republic	Denmark	Dominican Republic
Ecuador	Egypt, Arab Rep.	El Salvador	Finland	France
Germany	Greece	Hungary	Iceland	Indonesia
Ireland	Italy	Japan	Korea, Rep.	Lebanon
Malaysia	Mexico	Morocco	Netherlands	New Zealand
Nigeria	Norway	Pakistan	Panama	Peru
Philippines	Poland	Portugal	Russian Federation	South Africa
Spain	Sweden	Thailand	Tunisia	Turkey
Ukraine	United Kingdom	United States	Uruguay	Venezuela, RB

The sources are as follows:

GDP Per Capita Growth: GDP per capita from World Development Indicators (WDI);

TFP: Pen World Table;

Consumption Share of GDP: calculated using final consumption expenditure and GDP data in WDI;

Net Foreign Asset to GDP Ratio: an updated dataset in Lane and Milesi-Ferretti (2007) (see http://www.philiplane.org/EWN.html).

B   Empirical Results for KM episodes

I use sudden stop episodes as in Korinek and Mendoza (2014) to show the persistent output-level effects of crises. One can see that this effect is robust to identification of crises. Furthermore, TFP displays a similar pattern to output, as in Figure 1.

Figure 17: Growth Rates in KM episodes (%)
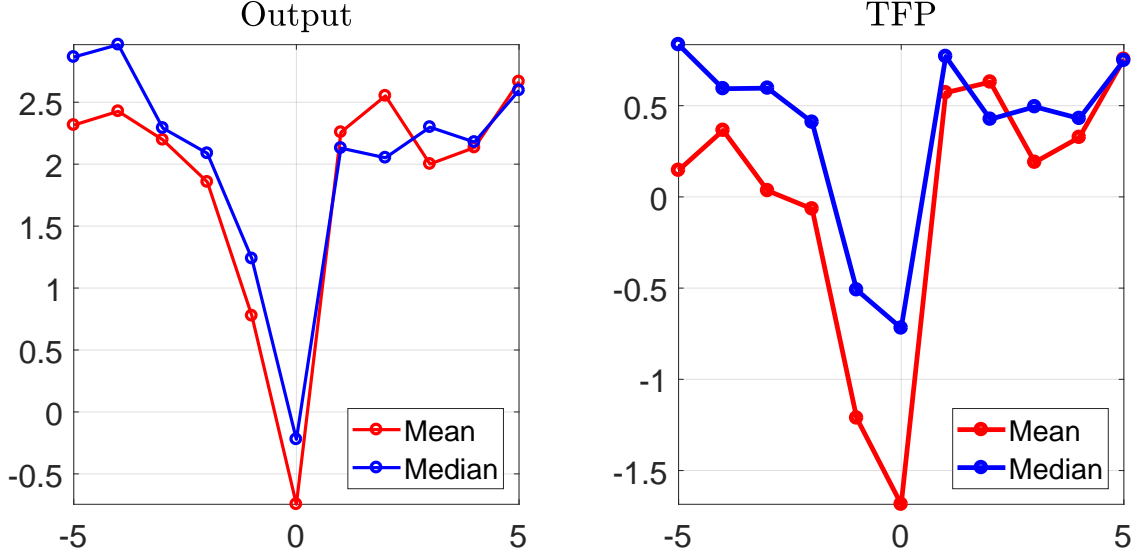
Note: The series are constructed using an 11-year window centering on the sudden stop episodes.

C   Normalized Economy

I normalize the economy by the endogenous variable z_t and denote normalized variables by a hat. The normalized competitive equilibrium conditions are given by

$$
\left(\hat{c}_t^h\right)^{-\gamma} \Psi_{1,t} = \beta g_{t+1}^{-\gamma} E_t \left[\left(\hat{c}_{t+1}^h\right)^{-\gamma} \left(\theta_{t+1} - h - \Psi_{2,t+1}\right) \right]
$$

$$
\left(\hat{c}_t^h\right)^{-\gamma} \hat{q}_t = \beta g_{t+1}^{1-\gamma} E_t \left[\left(\hat{c}_{t+1}^h\right)^{-\gamma} \left(\alpha \theta_{t+1} + \hat{q}_{t+1}\right) \right]
$$

$$
\left(\hat{c}_t^h\right)^{-\gamma} = \hat{\mu}_t^{CE} + \beta g_{t+1}^{-\gamma}(1+r) E_t \left[\left(\hat{c}_{t+1}^h\right)^{-\gamma} \right]
$$

$$
\hat{c}_t^h + \hat{\Psi}(g_{t+1}) + \hat{b}_{t+1} g_{t+1} = \theta_t - h + (1+r)\hat{b}_t
$$

$$
\hat{\mu}_t^{CE} \left(\hat{b}_{t+1} g_{t+1} + \phi \hat{q}_t \right) = 0, \text{with } \hat{\mu}_t^{CE} \geq 0.
$$

For the macroprudential social planner, the normalized equilibrium conditions are

$$\hat{\lambda}_t^{MP} = \left(\hat{c}_t^h\right)^{-\gamma} + \frac{\gamma\phi\hat{\mu}_t^{MP}\hat{q}_t}{\hat{c}_t^h} + \gamma\hat{\nu}_t^{MP}\left(\hat{c}_t^h\right)^{-\gamma-1}\Psi_{1,t}$$

$$\hat{\lambda}_t^{MP}\Psi_{1,t} - \frac{\phi\hat{\mu}_t^{MP}g_{t+1}^{-\gamma}\hat{G}_{1,t}}{\left(\hat{c}_t^h\right)^{-\gamma}} - \hat{\nu}_t^{MP}\left[g_{t+1}^{-1-\gamma}\hat{I}_{1,t} - \left(\hat{c}_t^h\right)^{-\gamma}\hat{\Psi}_{11,t}\right]$$

$$= \beta g_{t+1}^{-\gamma}E_t\left[\hat{\lambda}_{t+1}^{MP}\left(\theta_{t+1} - h - \Psi_{2,t+1}\right) - \hat{\nu}_{t+1}^{MP}\left(\hat{c}_{t+1}^h\right)^{-\gamma}\hat{\Psi}_{12,t+1}\right]$$

$$\hat{\lambda}_t^{MP} = \hat{\mu}_t^{MP} + \frac{\phi\hat{\mu}_t^{MP}g_{t+1}^{-\gamma}\hat{G}_{2,t}}{\left(\hat{c}_t^h\right)^{-\gamma}} + \hat{\nu}_t^{MP}g_{t+1}^{-1-\gamma}\hat{I}_{2,t} + \beta(1+r)g_{t+1}^{-\gamma}E_t\left[\hat{\lambda}_{t+1}^{MP}\right]$$

where

$$I(z_{t+1}, b_{t+1}) = z_{t+1}^{-\gamma}\hat{I}\left(\hat{b}_{t+1}\right),$$

$$I_{1,t} = (-\gamma)z_{t+1}^{-\gamma-1}\hat{I}\left(\hat{b}_{t+1}\right) + z_{t+1}^{-\gamma}\hat{I}'\left(\hat{b}_{t+1}\right)\frac{-b_{t+1}}{z_{t+1}^2} = -z_{t+1}^{-\gamma-1}\left[\gamma\hat{I} + \hat{I}'\hat{b}_{t+1}\right],$$

$$I_{2,t} = z_{t+1}^{-\gamma-1}\hat{I}'.$$

and

$$G(z_{t+1}, b_{t+1}) = z_{t+1}^{1-\gamma}\hat{G}\left(\hat{b}_{t+1}\right),$$

$$G_{1,t} = (1-\gamma)z_{t+1}^{-\gamma}\hat{G}\left(\hat{b}_{t+1}\right) + z_{t+1}^{1-\gamma}\hat{G}'\left(\hat{b}_{t+1}\right)\frac{-b_{t+1}}{z_{t+1}^2} = z_{t+1}^{-\gamma}\left[(1-\gamma)\hat{G} - \hat{G}'\hat{b}_{t+1}\right],$$

$$G_{2,t} = z_{t+1}^{-\gamma}\hat{G}'.$$

For the multi-instrument social planner, the normalized equilibrium conditions are

$$\hat{\lambda}_t^{MI} = \left(\hat{c}_t^h\right)^{-\gamma} + \frac{\gamma\phi\hat{\mu}_t^{MI}\hat{q}_t}{\hat{c}_t^h}$$

$$\hat{\lambda}_t^{MI}\Psi_{1,t} = \frac{\phi\hat{\mu}_t^{MI}g_{t+1}^{-\gamma}\hat{G}_{1,t}}{\left(\hat{c}_t^h\right)^{-\gamma}} + \beta g_{t+1}^{-\gamma}E_t\left[\hat{\lambda}_{t+1}^{MI}\left(\theta_{t+1} - h - \Psi_{2,t+1}\right)\right]$$

$$\hat{\lambda}_t^{MI} = \hat{\mu}_t^{MI} + \frac{\phi\hat{\mu}_t^{MI}g_{t+1}^{-\gamma}\hat{G}_{2,t}}{\left(\hat{c}_t^h\right)^{-\gamma}} + \beta(1+r)g_{t+1}^{-\gamma}E_t\left[\hat{\lambda}_{t+1}^{MI}\right]$$

D Proofs

D.1 Proof of Proposition 1

Proof. To implement the macroprudential social planner's allocation, I compare the normalized optimality conditions of private agents and of the macroprudential social planner (see Appendix C) and find that

$$
\tau_t^{MP,b} = \frac{\beta g_{t+1}^{-\gamma}(1+r) E_t \left[\gamma\phi\hat{\mu}_{t+1}^{MP}\hat{q}_{t+1} \left(\hat{c}_{t+1}^h\right)^{-1} + \gamma\hat{\nu}_{t+1}^{MP} \left(\hat{c}_{t+1}^h\right)^{-\gamma-1} \Psi_{1,t+1} \right]}{\left(\hat{c}_t^h\right)^{-\gamma}}
$$

$$
-\frac{\gamma\phi\hat{\mu}_t^{MP}\hat{q}_t \left(\hat{c}_t^h\right)^{-1} + \gamma\hat{\nu}_t^{MP} \left(\hat{c}_t^h\right)^{-\gamma-1} \Psi_{1,t} - \phi\hat{\mu}_t^{MP} g_{t+1}^{-\gamma}\hat{G}_{2,t} \left(\hat{c}_t^h\right)^{\gamma} - \hat{\nu}_t^{MP} g_{t+1}^{-1-\gamma}\hat{I}_{2,t}}{\left(\hat{c}_t^h\right)^{-\gamma}}
$$

D.2 Proof of Proposition 2

Proof. To implement the multi-instrument social planner's allocation, I compare the normalized optimality conditions of private agents and of the multi-instrument social planner (see Appendix C) and find that

$$
\tau_t^{MI,z} = \frac{\beta g_{t+1}^{-\gamma} E_t \left[\hat{c}_{t+1}^{-\gamma}\tau_{t+1}^{MI,z}\Psi_{2,t+1} + \gamma\phi\hat{\mu}_{t+1}^{MI}\hat{q}_{t+1} \left(\hat{c}_{t+1}^h\right)^{-1} \left(\theta_{t+1} - h - \Psi_{2,t+1}\right) \right]}{\Psi_{1,t} \left(\hat{c}_t^h\right)^{-\gamma}}
$$

$$
-\frac{\gamma\phi\hat{q}_t \left(\hat{c}_t^h\right)^{-1}\hat{\mu}_t^{MI}\Psi_{1,t} - \phi\hat{\mu}_t^{MI} \left(\hat{c}_t^h\right)^{\gamma} g_{t+1}^{-\gamma}\hat{G}_{1,t}}{\Psi_{1,t} \left(\hat{c}_t^h\right)^{-\gamma}},
$$

$$
\tau_t^{MI,b} = -\frac{\gamma\phi\hat{q}_t \left(\hat{c}_t^h\right)^{-1}\hat{\mu}_t^{MI} - \phi\hat{\mu}_t^{MI} \left(\hat{c}_t^h\right)^{\gamma} g_{t+1}^{-\gamma}\hat{G}_{2,t} - \beta g_{t+1}^{-\gamma}(1+r) E_t \left[\gamma\phi\hat{q}_{t+1} \left(\hat{c}_{t+1}^h\right)^{-1}\hat{\mu}_{t+1}^{MI} \right]}{\left(\hat{c}_t^h\right)^{-\gamma}}
$$

E Sensitivity Analysis

I conduct sensitivity analysis for different parameters in the model. As with the baseline calibration, I first give values for seven parameters, i.e., $\{\beta, \psi, r, \gamma, \alpha, \rho, \sigma\}$: I only change the value of one parameter while keeping the other parameter values the same, as in the baseline calibration. Given these values, I choose $\{\kappa, h, \phi\}$ to match average growth, the

consumption to GDP ratio, and the NFA-GDP ratio. I follow this strategy because I want the model to match average growth, which is affected by consumption's share of GDP and by the NFA-GDP ratio. The sensitivity analysis results are presented in Table 8, and I discuss the robustness of my results with respect to the parameters. One can see that the results do not change with α, since in the calibration, I assume that the collateral constraint binds in steady state, and that ϕ changes with α.

Impacts on Growth: The negative relationship between average growth and financial stability for the macroprudential social planner is very robust to all the parameter values. The multi-instrument social planner could generate a short-run growth spurt. In the baseline results, the growth spurt lasts for 18 years. I find that this number varies with different parameter values. Generally speaking, it is related to the welfare loss in the trend component of the consumption channel. The duration is longer if the multi-instrument social planner generates fewer welfare losses in this channel.

Welfare Gains: The results on welfare gains are robust to various parameters. In particular, I find that the macroprudential social planner can generate welfare gains equivalent to a 0.06 percent permanent increase in annual consumption, while the multi-instrument social planner can generate larger gains, equivalent to a 0.24 percent permanent increase in annual consumption. In particular, the size of gains increases with parameters that affect the size of externalities, such as ϕ. The gains also increase with parameters that make growth more sensitive to shocks, such as $\{\psi, \gamma\}$. Given that the social planners smooth the economy, welfare gains also increase with parameters that govern risk, such as $\{\rho, \sigma\}$.[27] The welfare gains are supposed to decrease with the discount rate β and the interest rate r, since they decide private agents' impatience condition, given by $\beta(1 + r)g^{-\gamma}$. Intuitively, when agents are more impatient, i.e., there is a lower β or r, the economy borrows more and ends up with more crises. Policy interventions should have more benefits, since they mitigate the frequency and severity of crises. Indeed, I find larger gains with a lower interest rate. However, I also find that welfare gains increase with β. This is because β decides the Euler equation of productivity. High β means that private agents care more about the reduction of growth during crisis. Hence, policy interventions can generate larger benefits by reducing this reduction.

Size of Interventions: In the baseline results, I find that the macroprudential social planner imposes a 1.28 percent capital flows tax, while the multi-instrument social planner imposes a 1.12 percent capital flows tax and a 1.00 percent subsidy on growth-enhancing

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[27]Here, lower ρ implies a higher risk for the economy, since it is more likely to enter a bad state tomorrow conditional on a good state today.

Table 8: Sensitivity Analysis

	Welfare Gains (%)						Tax on Capital Flows (%)				Subsidy on Growth (%)			Prob. of Crisis (%)			Average GDP Growth (%)			Reverse in Growth
	MP (overall)	MP (growth)	MP (consumption)	MI (overall)	MI (growth)	MI (consumption)	MP	MI (average)	MI (ex-ante)	MI (ex-post)	MI (average)	MI (ex-ante)	MI (ex-post)	CE	MP	MI	CE	MP	MI	MI
baseline	0.06	−0.34	0.40	0.24	−0.13	0.38	1.28	1.12	1.10	1.19	1.00	1.87	−1.78	6.23	1.89	14.23	2.315	2.307	2.289	17.51
β = 0.93	0.01	−0.04	0.05	0.09	−1.04	1.14	1.51	0.76	5.03	0.11	−1.40	4.27	−2.27	13.24	12.39	13.84	2.315	2.312	2.197	1.87
β = 0.95	0.03	−0.17	0.20	0.18	−0.82	1.01	1.67	1.17	2.95	0.23	−0.18	3.34	−2.04	10.83	9.52	13.66	2.318	2.312	2.240	3.39
ψ = 0.94	0.12	−0.48	0.59	0.36	−0.05	0.41	1.65	1.30	1.20	1.74	1.49	2.06	−1.14	2.86	1.89	15.33	2.323	2.311	2.292	23.22
ψ = 0.96	0.03	−0.16	0.18	0.15	−0.17	0.33	1.04	0.90	1.14	0.17	0.54	1.65	−2.94	7.32	2.06	12.36	2.308	2.305	2.287	13.16
φ = 0.07	0.01	−0.12	0.13	0.10	−0.19	0.29	0.81	0.80	1.19	−0.40	0.35	1.55	−3.27	7.27	6.66	11.56	2.308	2.306	2.289	11.00
φ = 0.08	0.02	−0.17	0.20	0.16	−0.17	0.33	0.94	0.93	1.15	0.23	0.60	1.71	−2.83	7.43	2.34	12.96	2.311	2.307	2.288	15.24
r = 3%	0.12	−0.56	0.69	0.49	−1.30	1.82	2.59	2.12	2.83	1.54	1.26	4.20	−1.11	7.84	6.26	14.29	2.336	2.312	2.238	5.29
r = 4%	0.10	−0.41	0.51	0.38	−0.74	1.14	1.92	1.23	1.63	0.35	0.80	2.52	−2.89	7.35	2.49	16.18	2.323	2.310	2.260	7.33
γ = 3	0.21	−1.13	1.40	1.42	−4.06	5.77	2.38	2.44	3.14	1.96	0.64	3.94	−1.59	10.49	7.00	15.33	2.363	2.352	2.246	5.28
γ = 4	0.43	−1.77	2.19	3.78	−14.20	21.19	3.03	3.90	7.22	3.29	−0.65	6.40	−1.94	12.12	10.65	16.24	2.392	2.382	2.149	3.84
α = 0.3	0.06	−0.34	0.40	0.24	−0.14	0.39	1.28	1.12	1.10	1.19	1.00	1.87	−1.78	6.23	1.89	14.23	2.315	2.307	2.288	17.19
α = 0.4	0.05	−0.34	0.40	0.23	−0.19	0.43	1.37	1.09	1.05	1.20	0.84	1.92	−2.10	6.23	1.89	14.23	2.315	2.307	2.288	17.18
ρ = 0.80	0.03	−0.31	0.33	0.25	−0.05	0.30	1.35	0.09	0.39	−2.38	0.19	0.72	−4.26	5.93	2.22	15.12	2.295	2.287	2.265	14.87
ρ = 0.90	0.02	−0.08	0.10	0.10	−0.51	0.62	0.93	0.96	1.81	0.56	0.40	2.63	−0.64	4.72	2.20	12.66	2.287	2.278	2.259	21.78
σ = 0.02														10.91	8.29	13.95	2.297	2.296	2.257	5.46
σ = 0.03	0.03	−0.19	0.22	0.17	−0.35	0.52	1.22	1.18	1.34	1.00	0.87	2.25	−0.75	7.38	6.75	15.72	2.303	2.300	2.270	9.14

Note: Welfare gains and taxes on debt are calculated by simulating the economy for 10,000 periods. Crises are defined as periods when the collateral constraint binds and the current account reversal exceeds 1 standard deviation of its long-run average.

expenditures. Furthermore, there is a difference when looking at the multi-instrument social planner's taxes from the ex-ante and the ex-post perspective. Generally speaking, the magnitude of the macroprudential capital flows tax varies with different parameters and depends on the size of externalities and the ergodic distribution of debt. For the multi-instrument social planner, it is a robust feature that she taxes borrowing and subsidizes growth-enhancing expenditures ex-ante. Ex-post, she always taxes growth-enhancing expenditures to relax the borrowing constraint and might also want to subsidize borrowing, depending on the tightness of the constraint. Hence, I find that it is not true that the multi-instrument social planner always imposes a lower ex-ante capital flows tax than the macroprudential social planner. In some cases, she actually chooses a much higher tax on borrowing, reflecting a stronger precautionary motive.

F   Numerical Methods for Solving Policy Functions

I first create a grid space $\mathcal{G}_b = \{\hat{b}^0, \hat{b}^1, \cdots\}$ for the bond holding $\hat{b}_t$ and a grid space $\Theta = \{\theta_1, \cdots, \theta_5\}$ for the exogenous technology shock θ_t. The discretization method for the log AR (1) process of θ_t follows the Rouwenhorst method, as in Kopecky and Suen (2010). I apply the endogenous gridpoint method as in Carroll (2006) to iterate first-order conditions in CE, MP, and MI, and the iteration stops until policy functions converge. Policy functions in competitive equilibrium include consumption $\mathcal{C}(\hat{b}_t, \theta_t)$, endogenous growth $\mathcal{G}(\hat{b}_t, \theta_t)$, asset price $\mathcal{Q}(\hat{b}_t, \theta_t)$, and bond holding $\mathcal{B}(\hat{b}_t, \theta_t)$. Denote the iteration step by j and start from arbitrary policy functions $\mathcal{C}^0(\hat{b}_t, \theta_t)$, $\mathcal{G}^0(\hat{b}_t, \theta_t)$, $\mathcal{Q}^0(\hat{b}_t, \theta_t)$, and $\mathcal{B}^0(\hat{b}_t, \theta_t)$, where 0 means the iteration step $j = 0$. Given policy functions in iteration step j, I solve policy functions for iteration $j + 1$ as follows:

1. For any $\theta_t \in \Theta$ and $\hat{b}_{t+1} \in \mathcal{G}_b$, I can solve $\{\hat{c}_t^h, g_{t+1}, \hat{q}_t\}$ using equilibrium conditions. Using the budget constraint, these allocations imply a unique $\hat{b}_t$. Then I have a combination of $\{\hat{b}_t\}$ and corresponding allocations $\{\hat{c}_t^h, g_{t+1}, \hat{q}_t, \hat{b}_{t+1}\}$. I can update policy functions using these combinations. In this process, I need to deal with the collateral constraint. Specifically, I assume that the constraint is slack and then check whether this condition is satisfied.

2. I first assume that the constraint is slack and allocations $g_{t+1}, \hat{c}_t^h, \hat{q}_t$ can be solved

using the following conditions:

$$\Psi_t(g_{t+1}) = \frac{E_t\left[\left(\mathcal{C}^j(\hat{b}_{t+1},\theta_{1+1})\right)^{-\gamma}\left(\theta_{t+1} - h - \Psi_2\left(\mathcal{G}^j(\hat{b}_{t+1},\theta_{t+1})\right)\right)\right]}{(1+r)E_t\left[\left(\mathcal{C}^j(\hat{b}_{t+1},\theta_{1+1})\right)^{-\gamma}\right]}$$

$$\hat{c}_t^h = g_{t+1}\left[\beta(1+r)E_t\left[\left(\mathcal{C}^j(\hat{b}_{t+1},\theta_{1+1})\right)^{-\gamma}\right]\right]^{-\frac{1}{\gamma}}$$

$$\hat{q}_t = \left(\hat{c}_t^h\right)^\gamma \beta g_{t+1}^{1-\gamma} E_t\left[\left(\mathcal{C}^j(\hat{b}_{t+1},\theta_{1+1})\right)^{-\gamma}\left(\alpha\theta_{t+1} + \mathcal{Q}(\hat{b}_{t+1},\theta_{t+1})\right)\right]$$

3. If the collateral constraint $-\hat{b}_{t+1}g_{t+1} \leq \phi\hat{q}_t$ is satisfied, I proceed to solve $\hat{b}_t$ using the budget constraint:

$$\hat{b}_t = \frac{\hat{c}_t^h + h + \hat{\Psi}(g_{t+1}) + \hat{b}_{t+1}g_{t+1} - \theta_t}{1+r}$$

4. If the constraint is violated, I can solve allocations $\{\hat{q}_t, \hat{c}_t^h, g_{t+1}\}$ using the following equations:

$$\left(\hat{c}_t^h\right)^{-\gamma}\Psi_t(g_{t+1}) = \beta g_{t+1}^{-\gamma}E_t\left[\left(\mathcal{C}^j(\hat{b}_{t+1},\theta_{1+1})\right)^{-\gamma}\left(\theta_{t+1} - h - \Psi_2\left(\mathcal{G}^j(\hat{b}_{t+1},\theta_{t+1})\right)\right)\right]$$

$$-\hat{b}_{t+1}g_{t+1} = \phi\hat{q}_t$$

$$\hat{q}_t = \left(\hat{c}_t^h\right)^\gamma \beta g_{t+1}^{1-\gamma}E_t\left[\left(\mathcal{C}^j(\hat{b}_{t+1},\theta_{1+1})\right)^{-\gamma}\left(\alpha\theta_{t+1} + \mathcal{Q}(\hat{b}_{t+1},\theta_{t+1})\right)\right]$$

5. I can update policy functions using the combinations of $\hat{b}_t$ and $\{g_{t+1}, \hat{c}_t^h, \hat{q}_t, \hat{b}_{t+1}\}$.

6. I keep iterating until policy functions in two consecutive iterations are close enough.

To solve policy functions for the two social planners, I need to solve additional policy functions of Lagrangian multipliers, such as $\mu(\hat{b}_t, \theta_t)$ and $\nu(\hat{b}_t, \theta_t)$, using equilibrium conditions described in Appendix C. Otherwise, the procedure is the same as above.