AMAT 342 Homework 1. Due Sept. 5, 2019

Do the following problems:

The *power set* $\mathcal{P}(S)$ of a set S is the set consisting of all subsets of S.
1a. What is $\mathcal{P}(\{1,2\})$?
1b. How many elements are in $\mathcal{P}(\mathcal{P}(\{1,2\}))$? Justify your answer.
1c. Prove that if S has infinitely elements, then so does $\mathcal{P}(S)$. (A one-line proof is sufficent.)

2a. Draw a diagram illustrating the set $\{1,2,3,4\}^2 \subset \mathbb{R}^2$.
2b. Draw a diagram illustrating the set $\{1,2,3,4\} \times I \subset \mathbb{R}^2$.
2c. Draw a diagram illustrating the set $I \times \{1,2,3,4\} \subset \mathbb{R}^2$.
2d. Let $S^1 \subset \mathbb{R}^2$ be the unit circle, i.e., the set of all points of distance 1 from the origin in $\mathbb{R}^2$. Draw a diagram illustrating the set $S^1 \times I \subset \mathbb{R}^3$. What shape is $S^1 \times I$?

If S is a finite set with n elements,
3a. how many elements does $\mathcal{P}(S^2)$ have?
3b. how many elements does $(\mathcal{P}(S))^2$ have?

For each function, give its image, and say whether the function is an injection, surjection, or bijection. If the function is a bijection, also give its inverse.
4a. $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^3$
4b. $f : \mathbb{R} \to \mathbb{R}^2$, $f(x) = (x, x^3)$
4c. $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^4$
4d. $f : \mathbb{R} \to [0,\infty)$, $f(x) = x^4$
4e. $f : \mathbb{R} \to [-1,1]$, $f(x) = \cos x$

5. Give a bijection $f : (0,1] \to [1,\infty)$. What is the inverse of f?
5b. [For fun, will not be graded] Give a bijection $g : (0,1) \to [0,1]$.