AMAT 342 Homework 3. Due Sept. 26, 2019

1. Prove that if X is path connected and $f : X \to Y$ is continuous, then $\mathrm{im}(f)$ is path connected.

2. Give an example where X is not path connected, $f : X \to Y$ is continuous, and $\mathrm{im}(f)$ is path connected. [Don't just draw a picture, specify the function explicitly.]

3. Give an example where X is path connected, $f : X \to Y$ is not continuous, and $\mathrm{im}(f)$ is not path connected. [Again, don't just draw a picture, specify the function explicitly.]

4. Prove that if X has k path components and $f : X \to Y$ is a continuous surjectiction, then Y has at most k path components. [HINT: It is sufficient to show that there exists a surjection from $\pi_0(X)$ to $\pi_0(Y)$.]

5. For each of the following functions $d : \mathbb{R} \times \mathbb{R} \to [0, \infty)$, say whether d is a metric. Briefly explain your reasoning.

a. $d(x, y) = \begin{cases} 0 & \text{if x=y,} \\ 1 & \text{otherwise.} \end{cases}.$

b. $d(x, y) = 2|x - y|$

c. $d(x, y) = (x - y)^2$.

d. $d(x, y) = |x| + |y|$.

e. $d(x, y) = \frac{|x|}{1+|y|}$.