AMAT 342 Homework 5. Due Thursday Oct. 31, 2019

For the following two questions, recall that we define a rigid motion to be a function $\phi : \mathbb{R}^3 \to \mathbb{R}^3$ of the form $\phi = T_{\vec{v}} \circ R_A$ for some rotation R_A and translation $T_{\vec{v}}$.

1. Show that if $\phi : \mathbb{R}^3 \to \mathbb{R}^3$ is of the form $\phi = R_A \circ T_{\vec{v}}$, where R_A is a rotation and $T_{\vec{v}}$ is a translation, then ϕ is a rigid motion.

2. Use the result of the previous problem to show that the composition of two rigid motions is a rigid motion. [Hint: for any two square matrices A and B, $\det(AB) = \det(A)\det(B)$. Also, use the associativity of function composition.]

3. Recall from class that for M a metric space with metric d_M and $x \in M$, we defined a function $d^x : M \to \mathbb{R}$ by $d^x(y) = d_M(x, y)$. Prove that d^x is continuous.

4. Prove that a surjective isometry is a homeomorphism. (NOTE: When I stated this result in class, I forgot to mention that this requires surjectivity.) [HINT: To establish continuity, you will need to deal directly with the rigorous definition of continuity.

5. Prove that a subset S of a metric space is open if and only if it contains none of is boundary points.

6. Which of the following subsets S of $\mathbb{R}^2$ are open?

- $S = \{(1, 1)\}$,
- $S=$ the x-axis,
- $S = \{(x, y) \mid y - 1 < x < y + 1\}$,
- $S = \{(x, y) \mid y - 1 < x \leq y + 1\}$,
- $S = \{(x, y) \mid y - 1 \leq x \leq y + 1\}$,