AMAT 342 Homework 7. Due Tuesday, Dec. 3, 2019

1. Consider the equivalence relation $\sim$ on $S^1$ given as follows: $x \sim y$ iff $x = y$ OR $x, y \in \{(-1,0),(0,1)\}$. Intuitively, the quotient space $S^1/\sim$ is the topological space obtained by gluing $(-1,0)$ to $(1,0)$.

a. Sketch the quotient space $S^1/\sim$. (That is, sketch an embedding of $S^1/\sim$ into $\mathbb{R}^2$.)

b. Is $S^1/\sim$ a manifold? Explain your answer informally.

2. Let $T \subset \mathbb{R}^2$ be given by $T = (I \times \{1/2\}) \cup (\{1/2\} \times I)$. Let $S = \{(1/2,0),(1/2,1),(0,1/2),(1,1/2)\} \subset T$. Define an equivalence relation $\sim$ on $T$ by $x \sim y$ iff $x = y$ or $x, y \in S$.

a. Sketch $T$, $S$, and the quotient space $T/\sim$.

b. Describe informally, in words, how $T/\sim$ is obtained from $T$ via gluing. (A single sentence is sufficient.)

c. Is $T$ a manifold? Is the quotient space $T/\sim$ a manifold? Explain your answer informally.

3. Consider the triangle $T$ in $\mathbb{R}^2$ given by $T = \{(x,y) \mid y > 0,\, y < x + 1,\, y < -x + 1\}$.

a. Sketch $T$.

b. What is the boundary of $T$? Is $T$ an open subset of $\mathbb{R}^2$?

c. Is $T$ a 2-D manifold? Briefly explain your answer.

4. Prove that if $T$ is a discrete topological space (i.e., every subset of $T$ is open) and $\sim$ is any equivalence relation on $T$, then the quotient space $T/\sim$ is also discrete topological space. [Hint: It is a very short and straightforward proof.]