Elementary Topology

AMAT 342

Instructor: Michael Lesnick

https://www.albany.edu/∼ML644186/

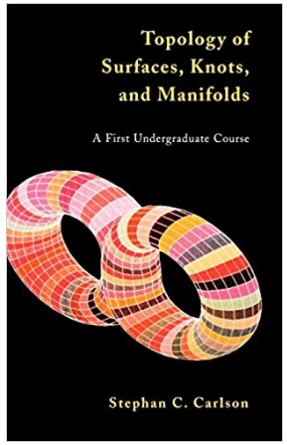
Today:

- Course logistics (syllabus)
- Introduction to topology
- Review of sets and functions.

Course webpage:
https://www.albany.edu/∼ML644186/TMAT_AMAT_342/index_342.html.

Required book:

- Topology of Surfaces, Knots, and Manifolds by Stefan Carlson.
- I will follow Carlson's text, sometimes detouring to treat a topic in more depth/rigor.



Other course materials:

- My (handwritten) lecture notes will be posted on online.
- Occasionally, there may be typed supplemental notes or xeroxed readings.

Homework:

- Due **Thursdays** most weeks.
- Posted on website roughly 1 week in advance.
- Homework grading (provisional plan):
  - 50% quick check for completeness,
  - 50% detailed grading of 1 or more "randomly" chosen problems.

HW collaboration:

- You may discuss with others,
- You must write up all assignments yourself,
- No late homework accepted.

The class will use the university's A-E grading scheme:

25%: HW and occasional Quizzes (equally weighted)

20%: Midterm I

20%: Midterm II

35%: Final

Lowest two HWs/quizzes will be dropped.

Exams may be curved (only upward).

Exam Dates:

Midterm I: Thursday October 10 (tentative),

Midterm II: Tue, November 12 (tentative),

Final: Tues. Dec 17th, 8:00 a.m. - 10:00.

Office Hours (Tentative):

- Tuesday 4:15-5:15, Friday 1:00-2:00, and by appointment.

Please let me know if you can't make these office hours.

Official Prerequisites:

- AMAT 214,
- AMAT 220,
- AMAT 299.

NOTE: Topology is a rewarding subject, but a challenging one.

- This class will require significant work outside of class each week: Careful reading, homework, **thinking**.

Miscellany

Class Rules:

- SUNY Albany's Undergraduate Academic Regulations apply to this course.

Miscellany

Class Rules:

- SUNY Albany's Undergraduate Academic Regulations apply to this course.

Course feedback:

- Constructive course feedback is welcome, either anonymously or 1-1, in person,
- Link to anonymous suggestion box is on course website.

Introduction to Topology

What is Topology?

The study of properties of shapes that are preserved under "continuous deformations."

What is Topology?

The study of properties of shapes that are preserved under "continuous deformations."

Continuous deformations:

- bending
- twisting
- stretching
- (but not tearing, puncturing, or gluing)

Topology is sometimes called "rubber sheet geometry."

More about topology:

- One of the major subfields of mathematics,
- Plays a central role in virtually all of modern mathematics,
- Also has many applications: physics, biology (DNA structure), economics, data science/machine learning,
- A very active area of research.

This course is primarily about foundational ideas of topology, not applications.

What is Topology?

The study of properties of shapes that are preserved under "**continuous deformations**."

What is Topology?

The study of properties of shapes that are preserved under "**continuous deformations**."

Continuous deformations:

- bending
- twisting
- stretching
- (but not tearing, puncturing, or gluing)

Topology is sometimes called "rubber sheet geometry."

What is Topology?

The study of properties of shapes that are preserved under "**continuous deformations**."

Continuous deformations:

- bending
- twisting
- stretching
- (but not tearing, puncturing, or gluing)

Topology is sometimes called "rubber sheet geometry."

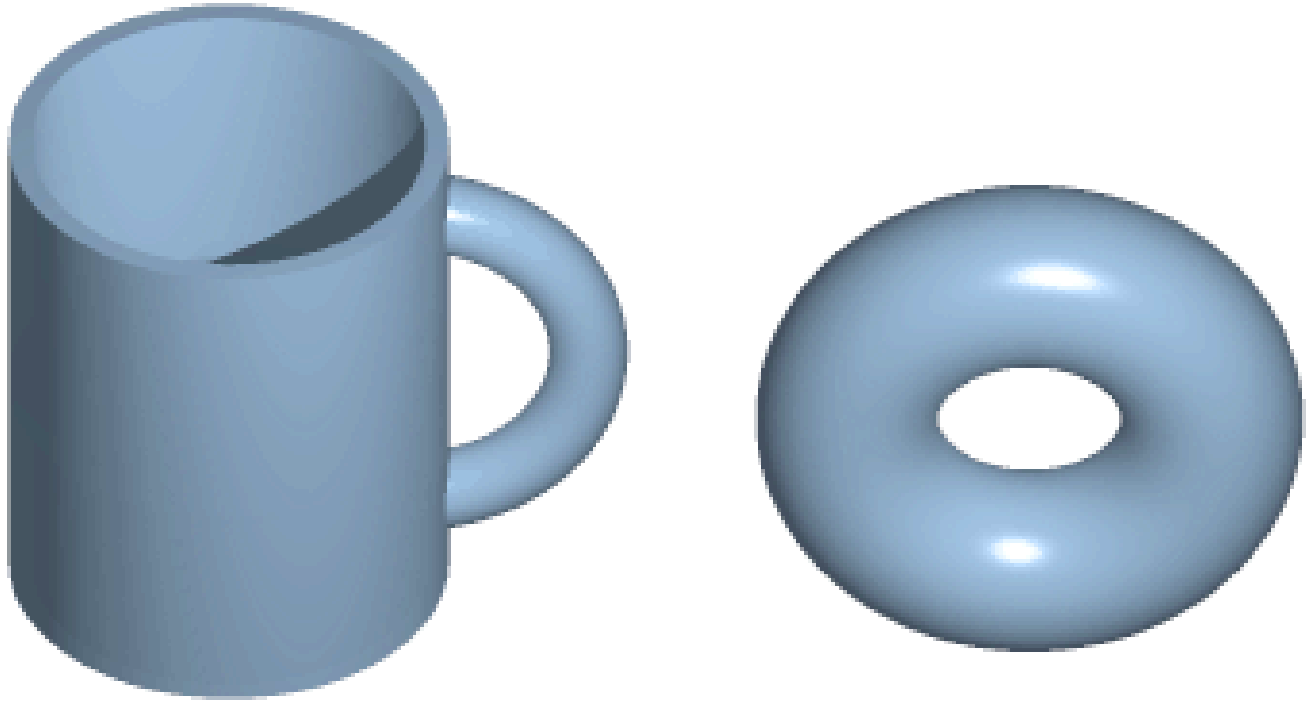
In topology, two shapes that can be continuously deformed into one another are thought of as "the same."

continuous deformations

Example: Consider the shapes of capital letters.

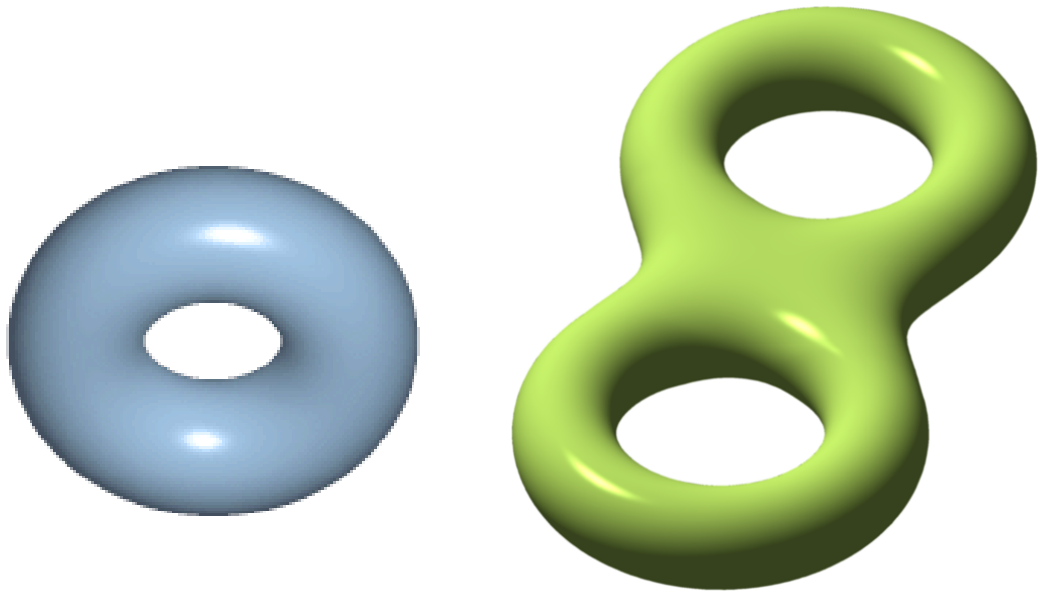
- **R** can be continuously deformed to **A** (https://vimeo.com/72136288)
- **Y** can be continuously deformed to **T**.
- **O** cannot be continuously deformed to **U**.

Example: A coffee mug can be continuously deformed into a donut.



https://en.wikipedia.org/wiki/Topology#/media/File:Mug_and_Torus_morph.gif

Example: A donut cannot be continuously deformed into a two-holed donut:



We'll formally define "continuous deformations" soon.

In fact we'll discuss two definitions:

- homeomorphism
- isotopy

What is Topology?

The study of properties of shapes that are preserved under "continuous deformations."

Continuous deformations:

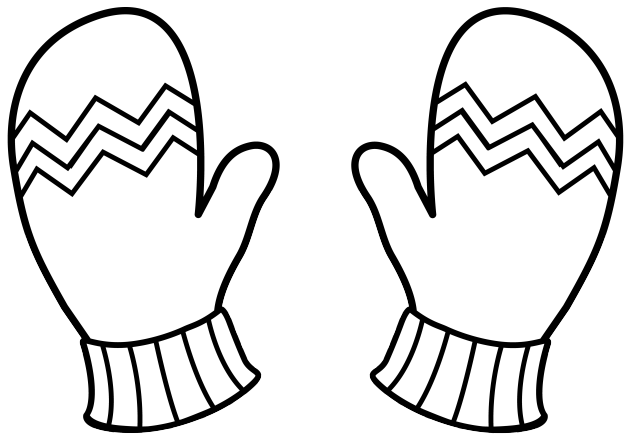
- bending
- twisting
- stretching
- (but **not** tearing, puncturing, or gluing)

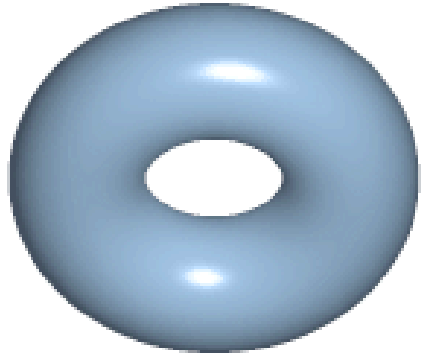
Topology is sometimes called "rubber sheet geometry."

What properties of shapes are preserved by continuous deformation?

Here are a few:

- Number of components
- Presence and number of holes





- Number of points where at least three curves curves meet.

X

What properties of shapes are **not** preserved by continuous deformation?

- lengths,
- angles,
- corners/edges.

What properties of shapes are **not** preserved by continuous deformation?

- lengths,
- angles,
- corners/edges.

These properties are important in mathematics, but are **systematically disregarded** in topology.

Subareas of Topology

Point-set topology

- Foundational technical concepts,
- Needed in analysis, differential geometry, and other advanced math,
- Some universities offer a full semester undergrad course in this.

Algebraic topology

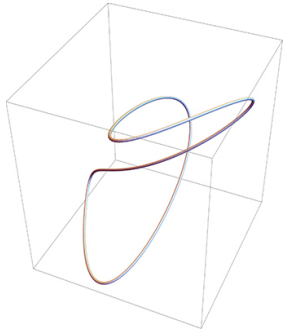
- Studies holes in shapes with the help of abstract algebra (groups, rings, vector spaces)

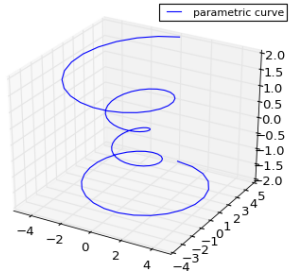
Topology of Manifolds

- Studies topological properties of curves, surfaces and their higher-dimensional generalizations.

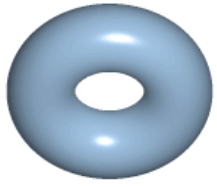
Curves, Surfaces, and Manifolds

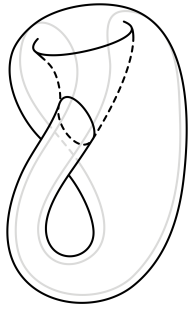
Curves are geometric objects that look like intervals near any point.

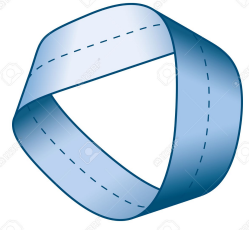




Surfaces are geometric objects that look like disks near any point.







Manifolds are n-dimensional generalizations of these.

And now, on to the details·....