AMAT 583 Homework 1. Due Sept. 12, 2019

The *power set* $\mathcal{P}(S)$ of a set S is the set consisting of all subsets of S.
1a. What is $\mathcal{P}(\{1,2\})$?
1b. How many elements are in $\mathcal{P}(\mathcal{P}(\{1,2\}))$? Justify your answer.

2a. Draw a diagram illustrating the set $\{1,2,3,4\}^2 \subset \mathbb{R}^2$.
2b. Draw a diagram illustrating the set $\{1,2,3,4\} \times I \subset \mathbb{R}^2$.
2c. Draw a diagram illustrating the set $I \times \{1,2,3,4\} \subset \mathbb{R}^2$.
2d. Let $S^1 \subset \mathbb{R}^2$ be the unit circle, i.e., the set of all points of distance 1 from the origin in $\mathbb{R}^2$. Draw a diagram illustrating the set $S^1 \times I \subset \mathbb{R}^3$. What shape is $S^1 \times I$?

If S is a finite set with n elements,
3a. how many elements does $\mathcal{P}(S^2)$ have?
3b. how many elements does $(\mathcal{P}(S))^2$ have?

For each function, give its image, and say whether the function is an injection, surjection, or bijection. If the function is a bijection, also give its inverse.
4a. $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^3$
4b. $f : \mathbb{R} \to \mathbb{R}^2$, $f(x) = (x, x^3)$ [NOTE: For this subproblem, there's not a slick way to write the image; just give $\mathrm{im}(f)$ in the obvious way using the curly bracket notation.
4c. $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^4$
4d. $f : \mathbb{R} \to [0, \infty)$, $f(x) = x^4$
4e. $f : \mathbb{R} \to [-1, 1]$, $f(x) = \cos x$

5. Let $f : R^2 \to \mathbb{R}$ be given by $f(x,y) = x + y$. For S the square

$$S = \{(x,y) \in \mathbb{R}^2 \mid -1 \leq x, y \leq 1,\}$$

describe $f(S)$ as an interval.

6a. Give a bijection $f : (0,1] \to [1, \infty)$. What is the inverse of f?
6b. Give a bijection $\mathbb{N} \cup \{a, b\} \to \mathbb{N}$.
6c. [Bonus, won't be graded] Building on your solution to 5b., describe a bijection $g : (0,1) \to [0,1]$.
(Hint: there is a bijection from the set $\{\frac{1}{2^n} \mid n \in \mathbb{N},\ n \geq 1\}$ to $\mathbb{N}$.)