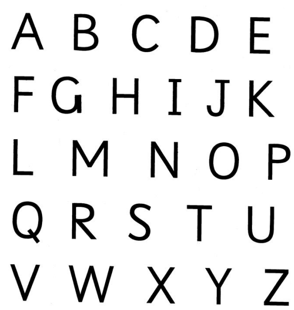
AMAT 583 Homework 2. Due Sept. 26, 2019

Do the following problems:

1. Considering the capital letters as unions of curves with no thickness, divide the letters into groups according to homeomorphism type.

For many letters, the homeomorphism type may depend on how they are written: Use the following for reference:



Note: The picture above doesn't really make clear what the homeomorphism type of the "no-thickness" version of the G or R would be. So don't include these in your grouping.

2. Repeat the exercise above, but with thickened versions of the letters above, as illustrated here:

To be, clear, you should consider the solid thick letters, not their boundaries. NOTE: You should consider R and Q here as well.

3. Let $L \subset \mathbb{R}^3$ be the lower hemisphere, i.e.

$$L = \{(x, y, z) \mid x^2 + y^2 + z^2 = 1 \text{ and } z \leq 0\}.$$

Let $D \subset \mathbb{R}^3$ be the closed disk in the xy-plane, i.e.,

$$D = \{(x, y, 0) \mid x^2 + y^2 \leq 1\}.$$

a. Give an explicit expression for a homeomorphism $f : L \to D$.

b. Give an explicit expression for f^{-1}.
c. Give an explicit isotopy from L to D.

HINT: The exercise in the Lecture 7 notes (not covered in class) is closely related and may be helpful. But first try solving this without looking at that.

4. For each of the following pairs of shapes S and T, sketch S and T, state whether they are homeomorphic. If they are homeomorphic, also say whether they are isotopic.
a. $S \subset \mathbb{R}^2$ is the union of two circles centered centered at the origin, of radii 1 and 2.
$T \subset \mathbb{R}^2$ is the union of two circles centered centered at the origin, of radii 3 and 4.

b. S as in the previous problem.
$T \subset \mathbb{R}^2$ is the union of a circle of radius 1 centered at the origin and a circle of radius 1 at centered at $(0, 3)$.

c. S as in the previous problem.
$T \subset \mathbb{R}^2$ is the union of a circle of radius 1 centered at the origin and a circle of radius 1 at centered at $(0, 1)$.

d. $S = \{(x, 0) \subset \mathbb{R}^2 \mid -1 \leq x \leq 1\}$.
$T = \{(0, y) \subset \mathbb{R}^2 \mid -1 \leq y \leq 1\}$.

5. For $S \subset \mathbb{R}^n$ and $x, y \in S$, a *path from x to y in S* is a continuous function $\gamma : I \to S$ with $\gamma(0) = x$ and $\gamma(1) = y$. S is called *path connected* if for any $x, y \in S$, there exists a path from x to y in S.

Prove that for homeomorphic spaces S and T, S is path connected if and only if T is path connected.

6 [Extra Credit] Using the rigorous definition of continuity, prove that if S is a finite set of points in $\mathbb{R}^m$ and $T \subset \mathbb{R}^n$ is any subset, any function $f : S \to T$ is continuous. HINT: Use that fact that for any $x \in S$, there is some $\delta > 0$ such that no other point of y is within distance δ of x.