AMAT 583 Homework 3. Due October 17, 2019

1. For each of the following relations $\sim$ on $\mathbb{Z}$, state whether $\sim$ is an equivalence relation. Explain your answer.

a. $x \sim y$ iff $x - y$ is divisible by 3. (Note: 0 is considered to be divisible by 3.)
b. $x \sim y$ iff $\frac{x}{y} = 1$,
c. $x \sim y$ iff $xy \geq 0$.
d. $x \sim y$ iff $x = y$ or $x = -y$.

2. Prove that if X is path connected and $f : X \to Y$ is continuous, then $\text{im}(f)$ is path connected. [Hint: You need to show that for any $a, b \in \text{im}(f)$, there exists a path $\gamma : I \to \text{im}(f)$ from a to b.]

3. Give an example where X is not path connected, $f : X \to Y$ is continuous, and $\text{im}(f)$ is path connected. [Don't just draw a picture, specify the function explicitly.]

4. Give an example where X is path connected, $f : X \to Y$ is not continuous, and $\text{im}(f)$ is not path connected. [Again, don't just draw a picture, specify the function explicitly.]

5. For each of the following functions $d : \mathbb{R} \times \mathbb{R} \to [0, \infty)$, say whether d is a metric. Briefly explain your reasoning.

a. $d(x,y) = \begin{cases} 0 & \text{if x=y,} \\ 1 & \text{otherwise.} \end{cases}$.
b. $d(x,y) = 2|x - y|$
c. $d(x,y) = (x - y)^2$.
d. $d(x,y) = |x| + |y|$.
e. $d(x,y) = \frac{|x|}{1+|y|}$.

6. Compute the edit distance between the following pairs of sequences:

1. $x = AAAA$, $y = AA$
2. $x = AAAA$, $y = AAAT$
3. $x = GTAA$, $y = TAAG$.
4. $x = GGGG$, $y = TT$.

7. Consider $P, Q \in O^2$. (That is, P and Q are each ordered pairs of points in $\mathbb{R}^3$). Find a simple formula for $RMSD(P, Q)$.

8. [Bonus] Show that for $\sim$ the equivalence relation on O^2 described in lecture, the metric space $(O^2/\sim, \overline{RMSD})$ is homeomorphic to a subset of $\mathbb{R}$ with its the usual Euclidean metric.

9. [Bonus] Show that if

$$P, P, Q, Q' \subset O^n$$

and $P = \phi(P')$, $Q = \psi(Q')$ for some rigid motions $\phi, \psi : \mathbb{R}^3 \to \mathbb{R}^3$, then

$$RMSD(P, Q) = RMSD(P', Q').$$

[Hint: It suffices to show both that

$$RMSD(P, Q) \leq RMSD(P', Q'),$$

and that

$$RMSD(P', Q') \leq RMSD(P, Q).$$

You may use without proof the fact that the composition two rigid motions is a rigid motion.]

10. [Optional exercise, not for credit, may not be graded] Check that $\overline{RMSD}$ is indeed a metric on $O^n/\sim$.