AMAT 583 Lecture 18

Today: Brief exam review
Some examples of gluing
Clustering

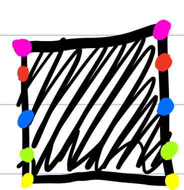
Gluing revisited

In an earlier lecture, I said I show some interesting examples of gluing constructions but then I forgot!

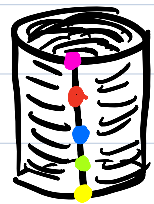
Before moving on to clustering, I want to give a few more famous examples of gluing constructions.

These can be specified formally by the quotient space construction we saw earlier, but I will be informal.

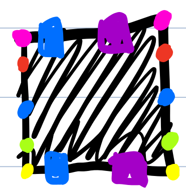
1. Recall the example from earlier



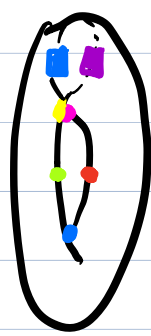
If we start with the square $I \times I$ and glue $(0,y)$ to $(1,y)$ $\forall y \in I$, we get the cylinder



2. What if we also glue the top edge to the bottom edge, i.e. glue

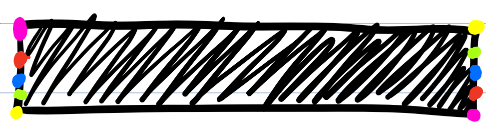


$(x,0)$ to $(x,1)$



We get a torus, i.e. surface of a donut.

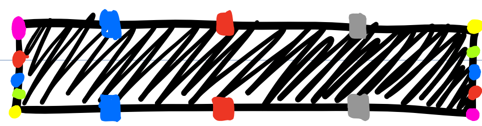
3. What if in the first example, we instead glue $(0,y)$ to $(1,1-y)$



We get the Mobius band

This is a surface with one side!

4. Suppose now that in the above example, we also glue the top edge to the bottom edge:

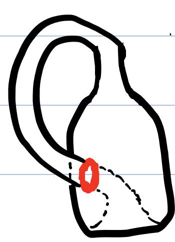


That is, we glue $(0,y)$ to $(1,1-y)$ $\forall y \in I$, and

$$(x,0) \text{ to } (x,1) \;\forall x \in I.$$

We get a surface K we call the Klein bottle.

Fact: K admits no embedding into $\mathbb{R}^3$. The figure below illustrates the image of a non-injective map $f: K \to \mathbb{R}^3$.



f is "almost injective"; all points in $\mathrm{im}(f)$ are mapped to by a unique point of K, except for the points in the red circle, which are hit by two points.

If we consider $j: \mathbb{R}^3 \to \mathbb{R}^4$, $j(x,y,z) = (x,y,z,0)$, then $j \circ f$ can be perturbed to an embedding: We can use the extra coordinate to perturb away the "self-intersection".

Thus, K embeds in $\mathbb{R}^4$.

Topogists love the Klein bottle because it is a surface w/ no boundary that is "<u>non-orientable</u>."

Informally, non-orientable means the surface has no separate inside and outside.