AMAT 584 Homework 1

Due Friday, February 14

February 19, 2020

1 Introduction

Problem 1. Which of the following point sets are in general position?

a. $\{(0,1),(1,3),(2,5)\}$,

b. $\{(0,0),(1,0),(2,4)\}$,

c. $\{(0,0),(0,1),(1,0),(1,1)\}$,

d. $\{(0,0,0),(0,1,0),(1,0,0),(1,1,0)\}$,

e. $\{(0,0,0),(0,1,0),(1,0,0),(1,1,1)\}$.

Problem 2. Which of the following sets is a (geometric) simplex? If the set is a simplex, give its dimension, and express it as the convex hull of a set of points in general position, using the bracket notation.

a. $\{(x,3x)\in\mathbb{R}^2 \mid 0\le x\le 1\}$,

b. $\{(x,y)\in\mathbb{R}^2 \mid 0\le x\le 1,\ 0\le y\le 2\}$,

c. $\{(x,3x,x)\in\mathbb{R}^3 \mid 0\le x\le 1\}$,

d. $\{(x,y)\in\mathbb{R}^2 \mid 0\le x\le 1\}$,

e. $\{(x,y)\in\mathbb{R}^2 \mid 0\le x,\ 0\le y\le 1-x\}$.

Problem 3. Which of the following sets of simplices is a geometric simplicial complex? For each, if the answer is no, explain which property fails; and if the answer is yes, give the dimension of the complex.

a. $\{[0],[0,1]\}$,

b. $\{[0],[1],[0,1]\}$,

c. $\{[0],[1],[2],[0,2]\}$,

d. $\{[(0,0)],[(0,1)],[(1,0)],[(0,0),(0,1),(1,0)]\}$,

e. $\{[(0,0)],[(0,1)],[(1,0)],[(1,1)],[(0,0),(0,1)],[(0,0),(1,0)],[(0,1),(1,0)]\}$,

f. $\{[(0,0)],[(0,1)],[(1,0)],[(1/4,1/4)],[(0,0),(0,1)],[(0,0),(1,0)],[(0,1),(1,0)]\}$,

Problem 4. Which of the following sets is an abstract simplical complex? For each, if the answer is no, explain why; and if the answer is yes, give the dimension of the complex, and sketch its geometric realization, up to homeomorphism.

a. $\{[a],[b],[a,b,c]\}$,

b. $\{[a],[b],[c],[a,b,c]\}$,

c. $\{[a],[b],[c],[a,b]\}$,

d. $\{[a],[b],[c],[d],[a,b],[c,d]\}$,

e. $\{[a],[b],[c],[d],[a,b],[b,c],[c,d],[a,d],[a,c],[a,b,c]\}$.

Problem 5. Let

$$X=\{[A],[B],[C],[A,B],[B,C],[A,C],[A,B,C]\} \quad Y=\{[A],[B],[C],[A,B],[B,C]\}.$$

- Let $f: V(X)\to V(Y)$ be given by $f(x)=x$ for all x. Does f define a simplicial map $f: X\to Y$? Briefly explain your answer.

Problem 6. For X as in the previous problem and W any abstract simplicial complex, explain why any map $f: V(W)\to V(X)$ defines a simplicial map $f: W\to X$.