AMAT 584 Homework 1 Solutions

1 Introduction

Problem 1. Which of the following point sets are in general position?

a. $\{(0,1),(1,3),(2,5)\}$,

b. $\{(0,0),(1,0),(2,4)\}$,

c. $\{(0,0),(0,1),(1,0),(1,1)\}$,

d. $\{(0,0,0),(0,1,0),(1,0,0),(1,1,0)\}$,

e. $\{(0,0,0),(0,1,0),(1,0,0),(1,1,1)\}$.

Answer: b. and e.

Problem 2. Which of the following sets is a (geometric) simplex? If the set is a simplex, give its dimension, and express it as the convex hull of a set of points in general position, using the bracket notation.

a. $\{(x,3x)\in\mathbb{R}^2 \mid 0\le x\le 1\}$, **Answer**: This is the 1-D geometric simplex $[(0,0),\ (1,3)]$

b. $\{(x,y)\in\mathbb{R}^2 \mid 0\le x\le 1,\ 0\le y\le 2\}$, **Answer**: Not a geometric simplex

c. $\{(x,3x,x)\in\mathbb{R}^3 \mid 0\le x\le 1\}$, **Answer**: This is the 1-D geometric simplex $[(0,0,0),(1,3,1)]$

d. $\{(x,y)\in\mathbb{R}^2 \mid 0\le x\le 1\}$, **Answer**: Not a geometric simplex.

e. $\{(x,y)\in\mathbb{R}^2 \mid 0\le x,\ 0\le y\le 1-x\}$. **Answer**: This is the 2-D geometric simplex $[(0,0),(1,0),(0,1)]$.

Problem 3. Which of the following sets of simplices is a geometric simplicial complex? For each, if the answer is no, explain which property fails; and if the answer is yes, give the dimension of the complex.

a. $\{[0],[0,1]\}$, **Answer**: No. $[1]$ is a face of $[0,1]$ but is not present.

b. $\{[0],[1],[0,1]\}$, **Answer**: Yes, 1-D.

c. $\{[0],[1],[2],[0,2]\}$, **Answer**: Yes, 1-D.

d. $\{[(0,0)],[(0,1)],[(1,0)],[(0,0),(0,1),(1,0)]\}$, **Answer**: No, e.g., $[[(0,0),,(0,1)]$ is a face of $[(0,0),(0,1),(1,0)]$, but is not present.

e. $\{[(0,0)],[(0,1)],[(1,0)],[(1,1)],[(0,0),(0,1)],[(0,0),(1,0)],[(0,1),(1,0)]\}$, **Answer**: Yes, 1-D.

f. $\{[(0,0)],[(0,1)],[(1,0)],[(1/4,1/4)],[(0,0),(0,1)],[(0,0),(1,0)],[(0,1),(1,0)]\}$, **Answer**: Yes, 1-D.

Problem 4. Which of the following sets is an abstract simplical complex? For each, if the answer is no, explain why; and if the answer is yes, give the dimension of the complex, and sketch its geometric realization, up to homeomorphism.

a. $\{[a],[b],[a,b,c]\}$, **Answer**: No, e.g., $[c] \subset [a,b,c]$ is not present.

b. $\{[a],[b],[c],[a,b,c]\}$, **Answer**: No, e.g., $[a,b] \subset [a,b,c]$ is not present.

c. $\{[a],[b],[c],[a,b]\}$, **Answer**: Yes, 1-D.

d. $\{[a],[b],[c],[d],[a,b],[c,d]\}$, **Answer**: Yes, 1-D

e. $\{[a],[b],[c],[d],[a,b],[b,c],[c,d],[a,d],[a,c],[a,b,c]\}$. **Answer**: Yes, 2-D.

Problem 5. Let

$$X = \{[A],[B],[C],[A,B],[B,C],[A,C],[A,B,C]\} \quad Y = \{[A],[B],[C],[A,B],[B,C]\}.$$

Let $f : V(X) \to V(Y)$ be given by $f(x) = x$ for all x. Does f define a simplicial map $f : X \to Y$? Briefly explain your answer.
Answer: No, because $f(\{A,B,C\}) = \{A,B,C\}$ is not a simplex in Y.

Problem 6. For X as in the previous problem and W any abstract simplicial complex, explain why any map $f : V(W) \to V(X)$ defines a simplicial map $f : W \to X$.
Answer: For any simplex $\sigma \in W$, $f(\sigma)$ is a simplex in X, because X contains every non-empty subset of its vertex set.

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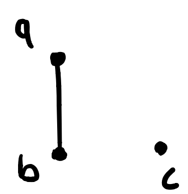

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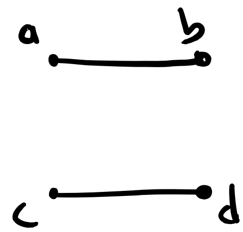

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