AMAT 584 Homework 2

Due Wednesday, February 26

Problem 1. 1. For each of the following abstract simplicial complexes, sketch the geometric realization (up to homoeomorphism) and compute the Euler characteristic:

a. $\{[a], [b], [c], [a, b]\}$,

b. $\{[a], [b], [a, b]\}$,

c. $\{[a], [b], [c], [d], [a, b], [c, d]\}$,

d. $\{[a], [b], [c], [d], [a, b], [b, c], [a, c], [c, d]\}$,

e. $\{[a], [b], [c], [a, b], [b, c], [a, c], [a, b, c]\}$.

Which pairs of these simplicial complexes have homotopy equivalent geometric realizations? Which pairs have equal Euler characteristics?

Problem 2. Let $X = \{(0, 0), (2, 0), (0, 1)\}$.

a. Give an explicit expression for $\check{\mathrm{C}}\mathrm{ech}(X, r)$ for each $r \geq 0$. (Here and forever after, use the closed-ball definition of $\check{\mathrm{C}}\mathrm{ech}(X, r)$.) HINT: To compute the value of r at which the 2-simplex $[(0, 0), (2, 0), (0, 1)]$ first appears in $\check{\mathrm{C}}\mathrm{ech}(V, r)$, it will be helpful to note that $x = (1, .5)$ is the midpoint of the line segment from (0,1) to $(2, 0)$, and

$$d(x, (0, 0)) = d(x, (2, 0)) = d(x, (0, 1)) = \frac{\sqrt{5}}{2}.$$

b. Give an explicit expression for $\mathrm{Rips}(X, r)$ for each $r \geq 0$.

c. The set $\mathrm{Vor}(X) = \{\mathrm{Vor}(x) \mid x \in X\}$ is called the *Voronoi decomposition of* X. Sketch $\mathrm{Vor}(X)$. In other words, sketch each of the Voronoi cells of X in a single diagram.

d. Give an explicit expression for $\mathrm{Del}(X, r)$ for each $r \geq 0$.

Problem 3. Let $X = \{(0, 0), (2, 0), (0, 2), (2, 2)\}$. Give an explicit expression for $\mathrm{Rips}(X, r)$ for each $r \geq 0$.

Problem 4. Prove that for any finite $X \subset \mathbb{R}^n$, $\mathrm{Rips}(X, r) \subset \check{\mathrm{C}}\mathrm{ech}(X, 2r)$. HINT: Use the triangle inequality.

Problem 5. Give an example of a finite set $X \subset \mathbb{R}^2$ and $0 \leq r < s$ such that $\mathrm{Rips}(X, r)$ is a connected graph and $\mathrm{Rips}(X, s)$ is a 4-dimensional simplicial complex.