AMAT 584 Homework 3

Due Friday, March 6

Problem 1. Compute the Euler characteristic curves of the Vietoris-Rips filtrations of the following sets $X \in \mathbb{R}^2$:

a. $X = \{(0,0),(2,0),(0,1)\}$,

b. $X = \{(0,0),(2,0),(0,2),(2,2)\}$.

HINT: In homework 2, you gave an explicit expression for the Vietoris-Rips filtration of each of these sets X. To avoid repeating that work, you can assume these as given.

Problem 2. Show that F_4 is not a field. HINT: Find a non-zero element in F_4 with no multiplicative inverse.

Problem 3. Let set S denote the set of all polynomials in one variable with real coefficients. For example,

$$3 + \frac{1}{2}x + 7x^2 + 4x^3 \in S.$$

S has a familiar definition of addition and multiplication. Moreover, S has an additive identity, the constant polynomial 0, and a multiplicative identity, the constant polynomial 1. Is S a field? Explain your answer.

Problem 4. Describe all subspaces of the following vector spaces:

a. F_2^2,

b. F_3^2.

Problem 5. Let V be a vector space over a field F. Racall that $\vec{0}$ denotes the additive identity of V, and 0 denotes the additive identity of F. Prove the following:

a. For all $a \in F$, $a\vec{0} = \vec{0}$.

b. For all $v \in V$, $0v = \vec{0}$.