

AMAT 584 Homework 5

Due Wednesday April 15

Problem 1. For each of the following simplicial complexes X

a. $X = \{[a], [b], [c], [d], [a, b], [c, d]\}$,

b. $X = \{[a], [b], [c], [d], [e], [a, b], [b, c], [c, d], [a, d], [a, c], [a, e], [b, e], [a, b, c]\}$,

do the following:

1. Sketch the simplicial complex.
2. Represent each non-zero boundary map ∂_j in the chain complex of X as a matrix with respect to the standard bases for $C_j(X)$ and $C_{j-1}(X)$. Use the given order on j -simplices.
3. Compute the dimension of each $Z_j(X)$, $B_j(X)$, and $H_j(X)$, for $j \geq 0$.

Problem 2. For X as in problem 1a., explicitly write down all of the cosets of $H_0(X)$. Sketch each element of each coset.

Problem 3. For X as in problem 1b., explicitly write down the coset of $H_1(X)$ containing each of the following elements of $Z_1(X)$, and sketch each element of the coset.

a. $z_1 = [a, b] + [b, c] + [a, c]$,

b. $z_2 = [c, d] + [a, d] + [a, c]$,

c. $z_3 = [a, b] + [a, e] + [b, e]$.

Problem 4. For both simplicial complexes considered in problem 1, give a basis for each non-zero $H_j(X)$. [Hint: $\{z_1, z_2, z_3\}$, as defined in the previous problem, is a basis for $Z_1(X)$.]