AMAT 584 — Takehome final, Spring 2020

Name: ____________________________

Instructions:

- The exam is due by email on May 15th at 11.59 p.m.
- Please show your work, but please make your written answers *as concise as possible.*
- You are not allowed to discuss the exam with *anyone* in any shape or form, except me. Any exam showing clear evidence of collaboration will be penalized heavily.

Problem 1 (6 points)**.** Which of the following sets is an abstract simplical complex? For each, if the answer is no, explain why; and if the answer is yes, give the dimension of the complex, sketch its geometric realization up to homeomorphism, and compute its Euler characteristic,

a. $\{[a], [b], [a, b], [b, c]\}$,

b. $\{[a], [b], [c], [a, c]\}$,

c. $\{[a], [b], [c], [a, b], [b, c], [a, c]\}$,

Problem 2 (2 points)**.** Give an example of a finite set $X \subset \mathbb{R}^2$ and $r \in [0, \infty)$ such that $\mathrm{VR}(X, r)$ is 3-dimensional and has two connected components.

Problem 3 (8 points)**.** Let $X = \{(0, 0), (2, 0), (1, \sqrt{3})\} \subset \mathbb{R}^3$. Regard X as a metric space with the Euclidean distance.

a. Compute the Vietoris-Rips filtration $\mathrm{VR}(X)$.

b. Compute and plot the Euler characteristic curve of $\mathrm{VR}(X)$.

c. Compute all persistent homology barcodes of $\mathrm{VR}(X)$; that is, compute $\mathrm{Barc}(H_j(\mathrm{VR}(X)))$ for all $j \geq 0$.

d. Compute all persistent homology barcodes of $\check{\mathrm{C}}\mathrm{ech}(X)$. [HINT: Let $\sigma = [(0, 0), (2, 0), (1, \sqrt{3})]$. Then $\sigma \in \check{\mathrm{C}}\mathrm{ech}(X, r)$ iff $r \geq \frac{2\sqrt{3}}{3}$. That is, $\mathrm{birth}(\sigma) = \frac{2\sqrt{3}}{3}$.]

Problem 4 (2 points)**.** Prove that for $f : V \to W$ any linear map, $\mathrm{im} f$ is a subspace of W.

Problem 5 (12 points)**.** For each of the following simplicial complexes X

a. $X = \{[a], [b], [c], [d], [a, b], [b, c], [a, c]\}$,

b. $X = \{[a], [b], [c], [d], [a, b], [b, c], [a, c], [c, d], [a, b, c]\}$, do the following:

1. Sketch the simplicial complex.
2. Represent each non-zero boundary map ∂_j in the chain complex of X as a matrix with respect to the standard bases for $C_j(X)$ and $C_{j-1}(X)$. Use the given order on j-simplices.
3. Compute the dimension of each $Z_j(X)$, $B_j(X)$, and $H_j(X)$, for $j \geq 0$.
4. Give a basis for each non-zero $H_j(X)$.

Problem 6 (4 points)**.** For X as in part a. of the previous problem, explicitly write down all of the cosets of $H_0(X)$, as in the posted solution to problem 2 of HW 5.

Problem 7 (2 points)**.** Suppose F is a filtration whose 1st persistent homology barcode is $\{[1,4),[2,6)\}$.

a. What is $\dim(H_1(F_3))$?

b. What is $\dim(H_1(F_5))$?