Master's Seminar

AMAT 680/681/682, Spring 2020

Instructor: Michael Lesnick

https://www.albany.edu/∼ML644186/

Course webpage (sylabus):

https://www.albany.edu/~ML644186/AMAT_680_Spring_2020/index_680.html.

Office Hours (Tentative):

- Monday, Wednesday 4:15-5:15, and by appointment.

Main course objective: Provide experience with

- doing independent math research,
- preparing math talks,
- presenting math to a live audience.

Main activity: Every student will give two presentations, to be followed by discussion.

Details about presentations:

- One presentation per week, on Mondays.
- Tentatively, these will start in three weeks (Monday Feb. 10).

Presentation Topics:

- **Suggested theme**: Computational mathematics,
- Topic to be chosen by student, with my input/consent,
- **Suggestion**: Each talk should mention at least one theorem and one algorithm,
- Should be outside of standard undergrad/masters math curriculum,
- No two presentations on same topic.

Presentation format:

- 30-40 minutes,
- Computer talks are encouraged,
- Blackboard talks are ok,
- Talks written in advance on a tablet/pen are ok, if prepared with care and good style,
- Introductory talks suitable for beginning grad. students in math,
- Substantive, yet engaging.

Logistics:

- Next week (Jan. 26 + 28), in place of the usual class, students will attend two math department colloquia on applied topology (ES-143, 3:00).
- The following week, we will meet on Monday (Feb. 3) and Wednesday (Feb. 5); I'll lecture on LaTeX/Beamer.

Starting Feb. 10, the week of the first presentation:

- We'll meet as a class on Mondays, but not Wednesdays,
- Each Wednesday, I'll meet individually with two students.
- 1 week before each talk, presenter will submit either a draft of slides or detailed notes.
- On both the Wednesdays **before and after** the presentation, I will meet with the presenter for 30 minutes during the regularly scheduled class time.

The class will use the university's S/U grading scheme.

Requirements for passing:

- Complete both talks, putting in a solid effort to prepare,
- Addend all classes ($>$2 absences without valid excuse = risk of failing),
- Participate in class discussions,
- Complete occasional additional assignments.

Class Rules:

- SUNY Albany's Undergraduate Academic Regulations apply to this course.

Most relevant to us is the regulations on **plagiarism**:

- If your talks closely follow one or more sources, or borrow figures/text from those sources, you must make that clear in your talk,
- Applies equally to books, websites, and youtube videos.

Feedback: Constructive course feedback is welcome.

Link to anonymous suggestion box is on course website.

Presentations will require substantial prep. time:

- identifying good resources,
- learning the topic,
- preparing slides/notes,
- grappling with software,
- practicing the presentation.

Pointers:

- Giving good talks is an acquired skill!
- Regardless of experience, good preparation is decisive.

Suggested topics (from the website):

- Persistent Homology,
- Mapper,
- Dimensionality reduction,
- The bootstrap method for computing confidence intervals
- Finite-state Markov Chains,
- PageRank,
- Network flow problems (graph theory),
- Matching problems (graph theory),
- Linear programming,
- The $P = NP$ problem in theoretical computer science,
- Spectral clustering.
- Fast Fourier transform
- Public key cryptography (e.g., RSA).
- Groebner bases.

Advertisement for a few of these topics

$P = NP$ problem:

- The most famous unsolved CS problem,
- 2^{nd} most famous open mathematics problem,

Rough outline:

- $P =$ computational problems that can be **solved** quickly.
  - Example: sorting a list
- $NP =$ computational problems for which candidate solutions can be **verified** quickly.
  - Example: finding the prime factorization of an integer.
- Many important problems are known to be in NP but not known to be in P,
- Are these classes of problems the same?

PageRank:

- The webpage ranking algorithm underlying Google's original search engine.
- Ranks the web using fundamental ideas from the theory of random walks (Markov chains)
- The computation amounts to an eigenvector computation.

Spectral clustering:

- **Clustering** is the problem of partitioning a set of points in $\mathbb{R}^n$ into groups so that nearby points are clustered together, while far away points are clustered apart.
- Traditional approaches do badly when distinct clusters are connected by a small "bridge."
- Spectral clustering uses an eigenvector computation to modify geometry of the data so that clustering can be performed more easily.

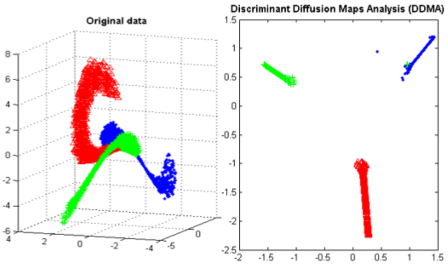

Figure from mathworks.com.

Solving systems of polynomial equations in several variables:

To determine whether the system

$$
\begin{aligned}
2x + 3y + 5z &= 7 \\
3x + y + 10z &= 2 \\
x + 3y + 3z &= 4
\end{aligned}
$$

has a solution, we can use linear algebra.

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has a solution, we can use linear algebra.

What about this system?:

$$\begin{aligned} 2x^2 + 3y^4 + 5z &= 7 \\ 3x^3 + y^5 + 10z^2 &= 2 \\ x + 3y + 3z^3 &= 4 \end{aligned}$$

If we allow for complex solutions, this can be solved in an elegant way using Groebner bases.

Fast Fourier Transform:

- Efficient algortihm for computing Fourier transform of a signal
- Based on linear algebra
- Ubiquitous in signal processing, and electronics.

Volunteer for first talk (February 10th)?

Homework (for Friday):

- (tentatively) choose a topic for first talk.
- Also make a 2nd and 3rd choice in case of conflict
- Send these to me by email.
- LaTeX/Beamer

About LaTeX/Beamer.