Section 19: hypothesis testing for paired samples

1. Does human body temperature change during the day? Listed below are body temperatures of subjects taken at two different times. (a) Do we have evidence at the 10%, 5%, 1% levels that body temperature changes between 8:00 am and 2:00 pm?

8:00 am	97	96.2	97.6	96.4	97.8	99.2
2:00 pm	98	98.6	98.8	98	98.6	97.6
	−1	−2.4	−1.2	−1.6	−0.8	1.6

$$\bar{d} = -0.9$$

$$\mu_d = \text{the mean difference of all people's body temps} = ?$$
$$s_d = 1.349074$$

$$H_0:\ \mu_d = 0$$
$$H_a:\ \mu_d \neq 0$$
$$t = \frac{\bar{d} - 0}{\left(\frac{s_d}{\sqrt{n}}\right)} = \frac{-0.9}{\left(\frac{1.349074}{\sqrt{6}}\right)} = -1.6341 \text{ with } df = 5$$

two tail	0.15	we're here	0.20
df 5	1.699		1.476

The *p*-value is between 0.15 and 0.20.

At the 10% level, *p*-value $> 0.15 > 0.10$ no evidence of H_a do not reject H_0

We do not reject the theory that body temp is the same; we find no evidence that it changes.

At the 5% level, *p*-value $> 0.15 > 0.05$ no evidence of H_a do not reject H_0

We do not reject the theory that body temp is the same; we find no evidence that it changes.

At the 1% level, *p*-value $> 0.15 > 0.01$ no evidence of H_a do not reject H_0

We do not reject the theory that body temp is the same; we find no evidence that it changes.

(b) Do we have evidence at these levels that body temperature goes up between these times?
Note that "morning – afternoon" trends negative. That is, morning trends lower. It may or may not be "suspiciously" lower, so it's a reasonable question.

one tail	0.075	we're here	0.1
df 5	1.699		1.476

The *p*-value is between 0.075 and 0.1.

At the 10% level, *p*-value < 0.10 evidence of H_a reject H_0

We do reject the theory that body temp is the same; we find evidence that it's higher later.

At the 5% level, *p*-value $> 0.075 > 0.05$ no evidence of H_a do not reject H_0

We do not reject the theory that body temp is the same; we find no evidence that it's higher later

At the 1% level, *p*-value $> 0.075 > 0.01$ no evidence of H_a do not reject H_0

We do not reject the theory that body temp is the same; we find no evidence that it's higher later

2. Is Friday the 13th unlucky? Researchers collected data for several hospitals on admissions resulting from motor vehicle crashes on two different days of the same month. (a) Do we have evidence that Friday the 13th is unlucky at the 10%, 5%, 1% levels?

Friday 6th	9	6	11	11	3	5
Friday 13th	13	12	14	10	4	12
	−4	−6	−3	1	−1	−7

$$\bar{d} = -3.33333$$

$$\mu_d = \text{the mean difference of all hospitals for these two weeks} = ?$$

$$s_d = 3.011091$$

$$H_0:\ \mu_d = 0$$
$$H_a:\ \mu_d < 0$$

$$t = \frac{\bar{d} - 0}{\left(\frac{s_d}{\sqrt{n}}\right)} = \frac{-3.33333}{\left(\frac{3.011091}{\sqrt{6}}\right)} = -2.7116 \text{ with } df = 5$$

one tail	0.01	we're here	0.025
df 5	3.365		2.571

The *p*-value is between 0.01 and 0.025.

At the 10% level, *p*-value $< 0.25 < 0.1$ evidence of H_a reject H_0

We reject the theory that Friday 13th is like any other day; we find evidence that it's unlucky.

At the 5% level, *p*-value $< 0.25 < 0.05$ evidence of H_a reject H_0

We reject the theory that Friday 13th is like any other day; we find evidence that it's unlucky.

At the 1% level, *p*-value > 0.01 no evidence of H_a do not reject H_0

We do not reject the theory that Friday 13th is like any other day; we find no evidence that it's unlucky.

(b) Do we have evidence that motor vehicle crash levels are different on Friday the 13th?

two tail	0.02	we're here	0.05
df 5	3.365		2.571

The *p*-value is between 0.02 and 0.05.

At the 10% level, p-value $< 0.05 < 0.1$ evidence of H_a reject H_0

We reject the theory that Friday 13th is like any other day; we find evidence that it's different.

At the 5% level, p-value < 0.05 evidence of H_a reject H_0

We reject the theory that Friday 13th is like any other day; we find evidence that it's different.

At the 1% level, p-value $> 0.2 > 0.01$ no evidence of H_a do not reject H_0

We do not reject the theory that Friday 13th is like any other day; we find no evidence that it's different.

3. Are people getting taller? Researchers studied the heights of several adult fathers and sons. Do we have evidence at the 10%, 5%, 1% levels that sons are taller than their fathers?

father height	70.3	67.1	70.9	66.8	72.8	70.4	71.8	70.1	69.9	70.8	70.2	70.4	72.4
son height	74.1	69.2	66.9	69.2	68.9	70.2	70.4	69.3	75.8	72.3	69.2	68.6	73.9
	−3.8	−2.1	4	2.4	3.9	0.2	1.4	0.8	−5.9	−1.5	1	1.8	−1.5

$$\bar{d} = -0.31538$$

$$\mu_d = \text{the mean difference of all possible father} - \text{son pairs} = ?$$

$$s_d = 2.897081$$

$$H_0:\ \mu_d = 0$$
$$H_a:\ \mu_d < 0$$

$$t = \frac{\bar{d} - 0}{\left(\frac{s_d}{\sqrt{n}}\right)} = \frac{= -0.31538}{\left(\frac{2.897081}{\sqrt{13}}\right)} = -0.3925 \text{ with } df = 12$$

one tail	0.25	we're here
df 12	0.695	

The p-value is more than 0.25.

At the 10% level, p-value $> 0.25 > 0.1$ no evidence of H_a do not reject H_0

We do not reject the theory that sons are the same height as their fathers; we find no evidence that they're taller.

At the 5% level, p-value $> 0.25 > 0.05$ no evidence of H_a do not reject H_0

We do not reject the theory that sons are the same height as their fathers; we find no evidence that they're taller.

At the 1% level, p-value $> 0.25 > 0.01$ no evidence of H_a do not reject H_0

We do not reject the theory that sons are the same height as their fathers; we find no evidence that they're taller.

4. Listed below are the heights (in inches) of candidates for president in recent times. For candidates who won more than once, only the first pair-up is listed. Does this sample provide evidence at the 10%, 5%, 1% levels that the taller candidate tends to win the election?

winner	71	74.5	74	73	69.5	71.5	75	72	70.5	69	74	70	71	72	70	67
runner-up	73	74	68	69.5	72	71	72	71.5	70	68	71	72	70	72	72	72
	−2	0.5	6	3.5	−2.5	0.5	3	0.5	0.5	1	3	−2	1	0	−2	−5

$$\bar{d} = 0.375$$

$$\mu_d = \text{the mean height difference of all winners vs. runnersup} = ?$$

$$s_d = 2.711088$$

$$H_0\text{:}\ \ \mu_d = 0$$
$$H_a\text{:}\ \mu_d > 0$$

$$t = \frac{\bar{d} - 0}{\left(\frac{s_d}{\sqrt{n}}\right)} = \frac{0.375}{\left(\frac{2.711088}{\sqrt{16}}\right)} = 0.5533 \ \text{ with } df = 15$$

one tail	0.25	we're here
df 12	0.691	

The *p*-value is more than 0.25.

At the 10% level, *p*-value $> 0.25 > 0.1$ no evidence of H_a do not reject H_0

We do not reject the theory that height makes no difference; we find no evidence that taller candidates win.

At the 5% level, *p*-value $> 0.25 > 0.05$ no evidence of H_a do not reject H_0

We do not reject the theory that height makes no difference; we find no evidence that taller candidates win.

At the 1% level, *p*-value $> 0.25 > 0.01$ no evidence of H_a do not reject H_0

We do not reject the theory that height makes no difference; we find no evidence that taller candidates win.

5. Are best actresses younger than best actors? Listed below are ages of actresses and actors at the times that they won Academy Awards. The data are paired according to the years that they won. Do we have evidence at the 10%, 5%, 1% levels that the "best actress" tends to be younger than the "best actor"?

best actress	28	32	27	27	26	24	25	29	41	40	27	42	33	21	35
best actor	62	41	52	41	34	40	56	41	39	49	48	56	42	62	29
	-34	-9	-25	-14	-8	-16	-31	-12	2	-9	-21	-14	-9	-41	6

$$\bar{d} = -15.6667$$

$$\mu_d = \text{the mean age difference of all possible best actor/actress pairs} = ?$$

$$s_d = 12.8767$$

$$H_0: \ \mu_d = 0$$
$$H_a: \ \mu_d < 0$$

$$t = \frac{\bar{d} - 0}{\left(\frac{s_d}{\sqrt{n}}\right)} = \frac{-15.6667}{\left(\frac{12.8767}{\sqrt{15}}\right)} = -4.7121 \ \text{ with } df = 14$$

one tail	we're here	0.0005
df 14		4.140

The p-value is less than 0.0005.

At the 10% level, p-value $< 0.0005 < 0.1$ evidence of H_a reject H_0

We reject the theory that their ages tend to be the same; we find evidence that best actress tends to be younger.

At the 5% level, p-value $< 0.0005 < 0.05$ evidence of H_a reject H_0

We reject the theory that their ages tend to be the same; we find evidence that best actress tends to be younger.

At the 1% level, p-value $< 0.0005 < 0.01$ evidence of H_a reject H_0

We reject the theory that their ages tend to be the same; we find evidence that best actress tends to be younger.

6. Are flights cheaper when scheduled earlier? Listed below are the costs (in dollars) of flights from New York to San Francisco for seven major airlines. Do we have evidence at the 10%, 5%, 1% levels that lights scheduled one day in advance cost more than flights scheduled 30 days in advance?

flight scheduled one day in advance	456	614	628	1088	943	567	536
flight scheduled 30 days in advance	244	260	264	264	278	318	280
	212	354	364	824	665	249	256

$$\bar{d} = 417.7143$$

$$\mu_d = \text{the mean difference of all possible airlines for the two scheduling dates} = ?$$

$$s_d = 234.5554$$

$$H_0:\ \mu_d = 0$$
$$H_a:\ \mu_d > 0$$

$$t = \frac{\bar{d} - 0}{\left(\frac{s_d}{\sqrt{n}}\right)} = \frac{417.7143}{\left(\frac{234.5554}{\sqrt{7}}\right)} = 4.7118 \text{ with } df = 6$$

one tail	0.0005	we're here	0.005
df 6	5.956		3.707

The *p*-value is between 0.0005 and 0.005.

At the 10% level, *p*-value $< 0.005 < 0.1$ evidence of H_a reject H_0

We reject the theory that scheduling date makes no difference; we find evidence that later scheduling costs more.

At the 5% level, *p*-value $< 0.005 < 0.05$ evidence of H_a reject H_0

We reject the theory that scheduling date makes no difference; we find evidence that later scheduling costs more.

At the 1% level, *p*-value $< 0.005 < 0.01$ evidence of H_a reject H_0

We reject the theory that scheduling date makes no difference; we find evidence that later scheduling costs more.

7. Do humans swim faster or slower in syrup? Twenty swimmers each swam a specified distance in a water-filled pool and in a pool where the water was thickened with food grade guar gum to create a syrup-like consistency. Their velocities in meters/sec are recorded. (a) Do we have evidence at the 10%, 5%, 1% levels that swimming speed changes?

Water	0.9	0.92	1	1.1	1.2	1.25	1.25	1.3	1.35	1.4
Syrup	0.92	0.96	0.95	1.13	1.22	1.2	1.26	1.3	1.34	1.41
	−0.02	−0.04	0.05	−0.03	−0.02	0.05	−0.01	0	0.01	−0.01

Water	1.4	1.5	1.65	1.7	1.75	1.8	1.8	1.85	1.9	1.95
Syrup	1.44	1.52	1.58	1.7	1.8	1.76	1.84	1.89	1.88	1.95
	−0.04	−0.02	0.07	0	−0.05	0.04	−0.04	−0.04	0.02	0

The average of the differences (water-syrup) is –0.004, and the standard deviation of the differences is 0.034702.

$$\bar{d} = -0.004$$

$$\mu_d = \text{the mean swimming difference of everyone in the two substances} = ?$$

$$s_d = 0.034702$$

$$H_0: \ \mu_d = 0$$
$$H_a: \ \mu_d \neq 0$$

$$t = \frac{\bar{d} - 0}{\left(\frac{s_d}{\sqrt{n}}\right)} = \frac{-0.004}{\left(\frac{0.034702}{\sqrt{20}}\right)} = 0.5155 \ \text{ with } df = 19$$

two tail	0.5	we're here
df 19	0.688	

The *p*-value is more than 0.5.

At the 10% level, *p*-value $> 0.5 > 0.1$ no evidence of H_a do not reject H_0

We do not reject the theory that syrup vs. water makes no difference in speed; we find no evidence that swimming speed changes in syrup.

At the 5% level, *p*-value $> 0.5 > 0.05$ no evidence of H_a do not reject H_0

We do not reject the theory that syrup vs. water makes no difference in speed; we find no evidence that swimming speed changes in syrup.

At the 1% level, *p*-value $> 0.5 > 0.01$ no evidence of H_a do not reject H_0

We do not reject the theory that syrup vs. water makes no difference in speed; we find no evidence that swimming speed changes in syrup.

(b) Do we have evidence at the 10%, 5%, 1% levels that swimming speed is faster in syrup?

one tail	0.25	we're here
df 19	0.688	

The *p*-value is more than 0.5.

At the 10% level, *p*-value $> 0.25 > 0.1$ no evidence of H_a do not reject H_0

We do not reject the theory that syrup vs. water makes no difference in speed; we find no evidence that swimming speed changes in syrup.

At the 5% level, *p*-value $> 0.25 > 0.05$ no evidence of H_a do not reject H_0

We do not reject the theory that syrup vs. water makes no difference in speed; we find no evidence that swimming speed changes in syrup.

At the 1% level, *p*-value $> 0.25 > 0.01$ no evidence of H_a do not reject H_0

We do not reject the theory that syrup vs. water makes no difference in speed; we find no evidence that swimming speed changes in syrup.

8. Do professional golfers play better in their first round? The scores of several professional golfers are recorded for their first and last rounds. Does the sample provide evidence at the 10%, 5%, 1% levels that professional golfers tend to play better (lower score) in their first round?

first round	66	70	64	71	65	71	71	71	71
last round	73	68	73	71	71	72	68	68	74
	-7	2	-9	0	-6	-1	3	3	-3

$$\bar{d} = -2$$

$$\mu_d = \text{the mean difference of all professional golfers for these two rounds} = ?$$

$$s_d = 4.5$$

$$H_0: \ \mu_d = 0$$
$$H_a: \ \mu_d < 0$$

$$t = \frac{\bar{d} - 0}{\left(\frac{s_d}{\sqrt{n}}\right)} = \frac{-2}{\left(\frac{4.5}{\sqrt{9}}\right)} = -1.3333 \text{with } df = 8$$

one tail	0.1	we're here	0.125
df 8	1.397		1.240

The *p*-value is between 0.10 and 0.125.

At the 10% level, *p*-value > 0.1 no evidence of H_a do not reject H_0

We do not reject the theory that golfers play the same in both rounds; we find no evidence that they play better in the first round.

At the 5% level, *p*-value $> 0.10 > 0.05$ no evidence of H_a do not reject H_0

We do not reject the theory that golfers play the same in both rounds; we find no evidence that they play better in the first round.

At the 1% level, *p*-value $> 0.10 > 0.01$ no evidence of H_a do not reject H_0

We do not reject the theory that golfers play the same in both rounds; we find no evidence that they play better in the first round.