
3.14 What is the probability of side effects from at least one agent?

a represents side effects from agent A
b represents side effects from agent B

You are told that the events are independent, so...

$P(a \cap b) = P(a) \times P(b)$

Use the Addition Law...

$P(a \cup b) = P(a) + P(b) - P(a \cap b)$
$P(a \cup b) = 0.10 + 0.20 - (0.10 \times 0.20) = 0.280$

3.15 What is the conditional probability that the father has influenza given that the mother has influenza?

3.16 What is the conditional probability that the father has influenza given that the mother does not have influenza?

You are told that in problem 3.12 that...

in 10% of families, MOTHER has influenza
in 10% of families, FATHER has influenza
in 2% of families, both MOTHER and FATHER have influenza

These are both CONDITIONAL PROBABILITY problems.

f represents the father with influenza
m represents the mother with influenza

Are these events (MOTHER with flu and FATHER with flu in the same family) INDEPENDENT? If they were...

$P(f \cap m) = P(f) \times P(m) = 0.10 \times 0.10 = 0.01$

But, you are told that...

$P(f \cap m) = 0.02$

So, the events are NOT INDEPENDENT and

$P(m \cap f) = P(m) \times P(f \mid m)$

The last term in the above equation is CONDITIONAL PROBABILITY, or what is the probability that the FATHER has the flu given that the MOTHER has the flu. Rearrange the equation...

$P(f \mid m) = P(m \cap f) / P(m)$
$P(f \mid m) = 0.02 / 0.10 = 0.20$

and...

$P(f \mid \bar{m}) = P(\bar{m} \cap f) / P(\bar{m})$

You are not given $P(\bar{m} \cap f)$ or $P(\bar{m})$ but you can calculate them.

Event 'f' can occur in two ways, with event 'm' and without event 'm'. Therefore...

$P(f) = P(m \cap f) + P(\bar{m} \cap f)$
$P(\bar{m} \cap f) = P(f) - P(m \cap f) = 0.10 - 0.02 = 0.08$

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and, $P(\bar{m})$ is the COMPLEMENT of P(m)...

$P(\bar{m}) = 1 - P(m) = 1 - 0.10 = .90$

So...

$P(f \mid \bar{m}) = P(\bar{m} \cap f) / P(\bar{m}) = 0.08 / 0.90 = 0.089$

Another way to do this is to imagine that there are 100 families...construct a 2x2 table. The 10% of mothers and fathers having the flu allow you to fill in the two of the marginal totals (the two 10s). The 2% of families with both the mother and father having the flu allow you to fill in the +/+ cell with a 2. Once these are filled in, you can complete the table...

		FATHER		
		+	-	TOTAL
	+	2	8	10
MOTHER	-	8	82	90
	TOTAL	10	90	100

The first ROW represents data CONDITIONAL on the MOTHER having the flu. The probability that the FATHER has the flu is...

$P(f \mid m) = 2 / 10 = 0.20$

The second ROW represents data CONDITIONAL on the MOTHER NOT having the flu. The probability that the FATHER has the flu is...

$P(f \mid \bar{m}) = 8 / 90 = 0.089$

3.17 What is the probability that all three individuals have Alzheimer's Disease?

77 represents the 77-year-old man who has Alzheimer's Disease
76 represents the 76-year-old woman who has Alzheimer's Disease
82 represents the 82-year-old woman who has Alzheimer's Disease

and the events are independent of one another.

$P(77 \cap 76 \cap 82) = P(77) \times P(76) \times P(82)$

Table 3.5 contains prevalence data that can be used as probabilities, for example...

the probability of a 77 year old male having Alzheimer's disease is 0.049 based on a prevalence for 75-79 year old males of 4.9 per 100 population

$P(77 \cap 76 \cap 82) = P(77) \times P(76) \times P(82) = 0.049 \times 0.023 \times 0.078 = 8.8 \times 10^{-5}$

3.18 What is the probability that at least one of the women has Alzheimer’s disease?

Use the Addition Law...

$P(76 \cup 82) = P(76) + P(82) - P(76 \cap 82)$
$P(76 \cup 82) = 0.023 + 0.078 - (0.023 \times 0.078) = 0.099$

The probability of one woman having Alzheimer's is INDEPENDENT of the other woman having Alzheimer's. That is why the last term in the above equation is based on...

$P(76 \cap 82) = P(76) \text{ x } P(82)$

3.19 What is the probability that at least one of the three individuals has Alzheimer’s disease?

What is the sample space...+ has Alzheimer's, - does not have Alzheimer's...

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     person
77    76    82
 +     +     +
 +     +     -
 +     -     +
 -     +     +
 +     -     -
 -     +     -
 -     -     +
 -     -     -
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Seven of the eight events in the sample space have at least one person with Alzheimer's. This problem is most easily answered by first computing the probability that NONE of the three individuals has Alzheimer's disease...

Pc is the complement of P, so Pc = 1 - P

Pc77 = 1 - 0.049 = 0.951
Pc76 = 1 - 0.023 = 0.977
Pc82 = 1 - 0.078 = 0.922

$P(c77 \cap c76 \cap c82) = Pc77 \text{ x } Pc76 \text{ x } Pc82 = 0.951 \text{ x } 0.977 \text{ x } 0.922 = 0.857$

Since the probability that NONE of the three individuals has Alzheimer's disease is 0.857, the probability that at least one person has Alzheimer's is...

$P(77 \cup 76 \cup 82) = 1 - 0.857 = 0.143$

3.20 What is the probability that exactly one of the three individuals has Alzheimer’s disease?

In the sample space shown in answer 3.19, there are three entries with exactly one person with Alzheimer's...

$P(\text{exactly } 1) = P(77 \cap c76 \cap c82) + P(c77 \cap 76 \cap c82) + P(c77 \cap c76 \cap 82)$
$P(\text{exactly } 1) = (0.049 \times 0.977 \times 0.922) + (0.951 \times 0.023 \times 0.922) + (0.951 \times 0.977 \times 0.078)$
$P(\text{exactly } 1) = 0.0441 + 0.0202 + 0.0725$
$P(\text{exactly } 1) = 0.137$

3.21 What is the conditional probability that the affected person is a woman given that we know that one of the individuals has Alzheimer’s disease?

In the previous problem, the problem that exactly one person has Alzheimer's is 0.137. That probability is calculated by adding...

77 year old man	$P(77 \cap c76 \cap c82) = (0.049 \times 0.977 \times 0.922) = 0.0441$
76 year old woman	$P(c77 \cap 76 \cap c82) = (0.951 \times 0.023 \times 0.922) = 0.0202$
82 year old woman	$P(c77 \cap c76 \cap 82) = (0.951 \times 0.977 \times 0.078) = 0.0725$

Of these events, two have only one woman. Therefore...

P (woman | exactly 1 affected) = (0.0202 + 0.0725) / 0.137 = 0.0927 / 0.137 = 0.676

One way to think about this answer is that specifying that exactly one person is affected creates a new sample space...

	person	
77	76	82
+	-	-
-	+	-
-	-	+

You know that each of these events has a probability in the original sample space...

	person		probability
77	76	82	
+	-	-	0.0441
-	+	-	0.0202
-	-	+	0.0725

Since this is now the entire sample space, the sum of the probabilities should add to 1. You can rescale the probabilities and make them add to 1 (but still have the same relationships to each other as within the original sample) space by dividing each by their sum in the original sample space (0.1368)...

	person		probability	rescaled probability
77	76	82		
+	-	-	0.0441	0.3223
-	+	-	0.0202	0.1477
-	-	+	0.0725	0.5300
	total		0.1368	1.0000

The probability that the affected person is woman is the sum of the rescaled probabilities for the 76 and 82 persons (the two woman)...0.1477 + 0.5300 = 0.678 (different from the 0.676 calculated above due to rounding in various calculations).

This is the same logic that is use din the next two problems (3.22 and 3.23).

3.22 What is the conditional probability that they are both women given we know that two of the three individuals have Alzheimer's disease?

In the sample space shown in answer 3.19, there are three entries with exactly two people with Alzheimer's...

77 year old man
and 76 year old woman $P(77 \cap 76 \cap c82) = (0.049 \times 0.023 \times 0.922) = 0.00104$
77 year old man
and 82 year old woman $P(77 \cap c76 \cap 82) = (0.049 \times 0.977 \times 0.078) = 0.00373$
76 year old woman
and 82 year old woman $P(c77 \cap 76 \cap 82) = (0.951 \times 0.023 \times 0.078) = 0.00171$

p(exactly 2) = .00104 + .00373 + .00171 = 0.00648

Of these events, only one has two woman. Therefore...

P (woman **|** exactly 2 affected) = 0.00171 / 0.00648 = 0.263

3.23 What is the conditional probability that both are less than 80 years, given we know that 2 of the 3 individuals have Alzheimer's disease?

In the previous problem, the problem that exactly two people have Alzheimer's is 0.00648, calculated by adding the probabilities of three events in the sample space. Of these three events, only one has both individuals less than 80 years of age...

77 year old man
and 76 year old woman $P(77 \cap 76 \cap c82) = (0.049 \times 0.023 \times 0.922) = 0.00104$

Therefore...

P (both < 80 years of age **|** exactly 2 affected) = 0.00104 / 0.00648 = 0.160

3.113 Is the proportion 100 out of 1000 a prevalence rate, an incidence rate, or neither?

This is an incidence rate because it counts the number of new cases in a given period of time.

3.114 ******** upon my review...this is an unfair question...there's not enough information in the chapter to answer this question ********

You are told that there are 100 new cases among 1000 Hispanic men and that 50 of the 100 new cases died.

Imagine that there are no men with heart attacks at the start of the study. At the end of 5 years, there would be 50 Hispanic men still alive who had a heart attack during the study period. Given 1000 men and 50 cases, the proportion of Hispanic men identified as having a heart attack would be 0.050.

You are also told that other studies show that if you start with 1000 non-Hispanic white men, 80 new cases would develop. The fatality rate among these cases is 20 per hundred, or 64 remain alive. Therefore, given 1000 men and 64 cases, the proportion of non-Hispanic white men identified as having a heart attack would be 0.064 (higher than the Hispanic proportion even though incidence is higher among Hispanic men).

3.115 The proportions are PREVALENCE. The proportions are based on surviving cases of CHD measured at one point in time. Since the case fatality rate is higher among Hispanics than among whites and prevalence is based solely on surviving cases, prevalence alone gives the incorrect impression that Hispanics are at lower risk for CHD.