Math 461: Topology, Fall 2009 — Quiz # 7, November 12

Name: ....

First Problem.

Let A and B be sets. Are the following statements true or false? Please circle your answers.

1] If there exists an injective function $f : \mathbb{N} \to A$, then A is uncountable .... TRUE | FALSE

2] If there exists an injective function $f : \mathbb{N} \to A$, then A is countable .... TRUE | FALSE

3] If there exists an injective function $f : \mathbb{N} \to A$, then A is infinite .... TRUE | FALSE

4] If A is infinite, then there exists an injective function $f : \mathbb{N} \to A$ .... TRUE | FALSE

5] A is countable if and only if there exists an injective function $f : \mathbb{N} \to A$ or $A = \emptyset$ .. TRUE | FALSE

6] A is countable if and only if there exists a surjective function $f : \mathbb{N} \to A$ or $A = \emptyset$ .. TRUE | FALSE

7] A is uncountable if and only if there exists an injective function $f : \mathbb{R} \to A$ .... TRUE | FALSE

8] A is uncountable if and only if there exists a surjective function $f : \mathbb{R} \to A$ .... TRUE | FALSE

9] A is uncountable if and only if there exists **no** surjective function $f : \mathbb{N} \times \mathbb{Z} \times \mathbb{Q} \to A$ TRUE | FALSE

10] If A is uncountable, then there exists a bijective function $f : A \to \mathbb{R}$, i.e., $A \simeq \mathbb{R}$ ... TRUE | FALSE

11] If A is countable and B is countable, then $A \cup B$ is countable .... TRUE | FALSE

12] If A is countable and B is uncountable, then $A \cup B$ is uncountable .... TRUE | FALSE

13] If A is uncountable and B is uncountable, then $A \cup B$ is uncountable .... TRUE | FALSE

14] If A is countable and B is countable, then $A \cap B$ is countable .... TRUE | FALSE

15] If A is countable and B is uncountable, then $A \cap B$ is countable .... TRUE | FALSE

16] If A is uncountable and B is uncountable, then $A \cap B$ is uncountable .... TRUE | FALSE

17] If $A \times B$ is countable, then both A and B are countable. .... TRUE | FALSE

18] If $A \times B$ is countable, then A or B is countable. .... TRUE | FALSE

19] If $A \times B$ is uncountable, then both A and B are uncountable. .... TRUE | FALSE

20] If $A \times B$ is uncountable, then A or B is uncountable. .... TRUE | FALSE

Second Problem.

Suppose that A is a set and that $B \subset A$ is a subset of A, and assume that $B \neq A$.
Is it possible for A and B to have the same cardinality, i.e., $A \simeq B$? .... ☐

A] No, it is not possible for any A.
B] Yes, it is possible for any A.
C] Yes, but only if A is empty.
D] Yes, but only if A is not empty.
E] Yes, but only if A is finite.
F] Yes, but only if A is infinite.
G] Yes, but only if A is countable.
H] Yes, but only if A is uncountable.