

## NNSE 508 EM

### Home assignment # 2

Due: February 17, 2014

1. Calculate a magnetic field of a thin long straight conductor with current  $I$  using both Bio-Savart law and Ampere's circuital law. Show that the results are the same.
2. Show that in the field of electromagnetic wave the magnetic part of the Lorentz force is smaller than the electric part.
3. EM wave with intensity  $1 \text{ mW/m}^2$  (in the air) falls normally on a seawater ( $\sigma = 4 \text{ S/m}$ ,  $\epsilon_r = 81$ ,  $\mu_r = 1$ ) surface.
  - a) Find propagation constant, attenuation constant, wavelength, and phase velocity in water and the amplitudes of  $E$  and  $H$  in the air for wave frequencies: 1 Hz, 1 kHz and 1 MHz.
  - b) At what depth the energy of these waves will drop by 10 times?
  - c) Is salt water a good conductor or good dielectric?
  - d) [bonus = 20 points extra] Find the amplitudes of  $E$  and  $H$  close to the surface but inside water for the same frequencies as in (a)

4. a) Show that if a cylindrical wire radius is much greater than skin depth,  $a \gg \delta$ , a resistance of wire per unit length is

$$R \approx \frac{1}{2\pi a \sigma \delta}$$

- b) Calculate an active resistance of a 1 km long 1 mm thick Cu wire at 60 Hz, 100 kHz and 100 MHz.

5. A plane EM wave at 100 MHz is propagating in a lossy material. The phase of the electric field shifts  $90^\circ$  over a distance of 0.5 m, and its peak value is reduced by 25% for each meter traveled. Find phase constant, attenuation constant and phase velocity of the wave.

6. (a) Show that for good conductors  $v_p = \sqrt{\frac{2\omega}{\mu\sigma}}$   $v_g = 2\sqrt{\frac{2\omega}{\mu\sigma}}$

- (b) show that for good dielectrics  $v_p = \frac{1}{\sqrt{\epsilon\mu}} \left[ 1 + \frac{1}{8} \left( \frac{\sigma}{\omega\epsilon} \right)^2 \right]^{-1}$   $v_g = \frac{1}{\sqrt{\epsilon\mu}} \left[ 1 - \frac{1}{8} \left( \frac{\sigma}{\omega\epsilon} \right)^2 \right]^{-1}$

Note that in (b), and possibly in (a), we have  $v_g > \frac{1}{\sqrt{\mu\epsilon}}$ , so over much of the spectrum a lossy dielectric is anomalously dispersive.