

Lecture contents

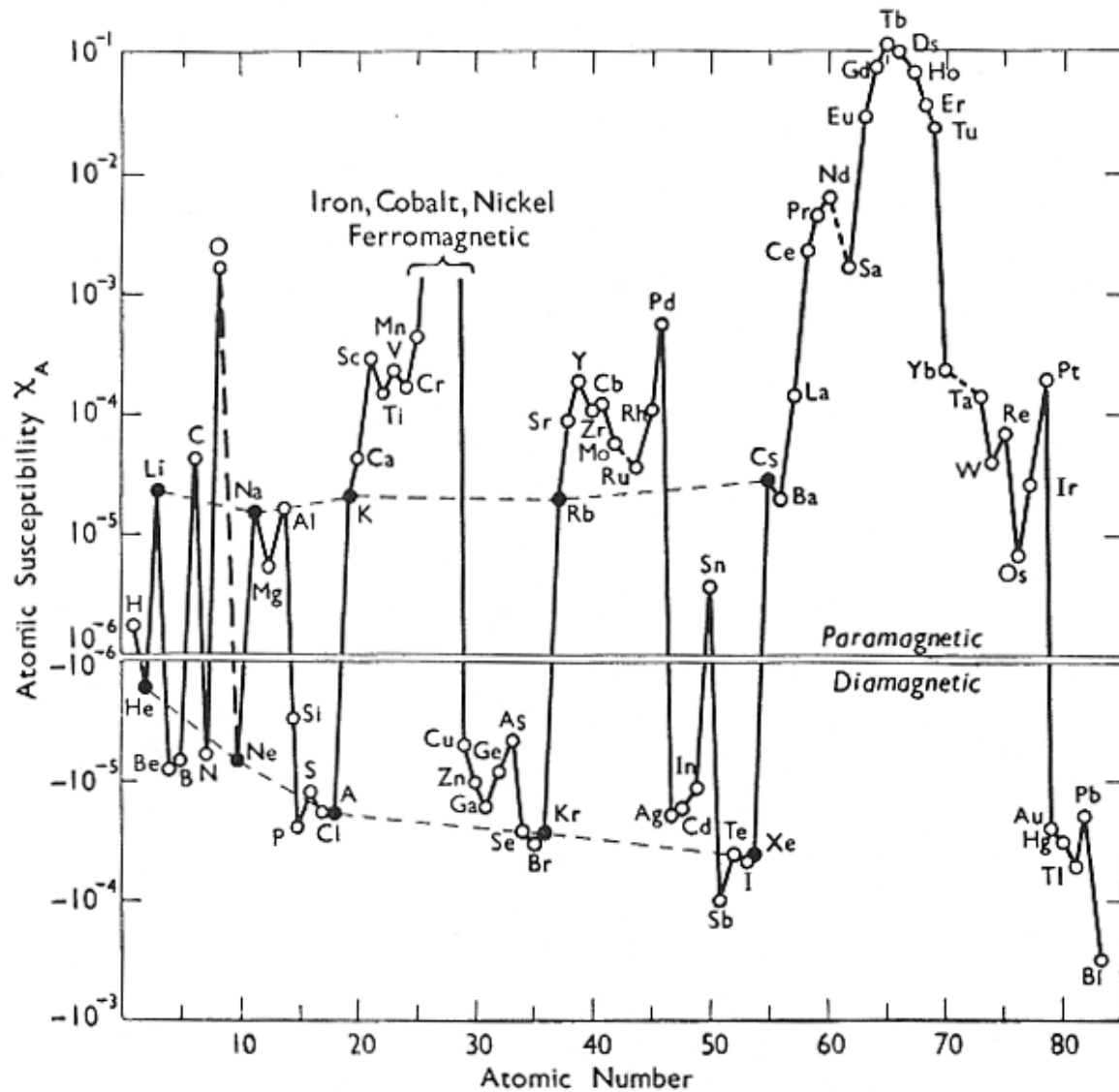
- Magnetic properties
- Diamagnetism
- Band paramagnetism
- Atomic paramagnetism
- Ferromagnetism
 - Molecular field theory
 - Exchange interaction

Magnetic properties of materials

• [SI]
$$\left\{ \begin{array}{l} \vec{B} = \mu_0 \vec{H} + \vec{M} \\ \vec{B} = \mu_0 \mu_R \vec{H} \\ \vec{M} = \chi \vec{H} \\ \mu_R = 1 + \chi \end{array} \right.$$

- M – magnetization or magnetic dipole density
- Diamagnetic $\chi \sim -10^{-5}$
- Paramagnetic $\chi \sim +10^{-5}$
- Ferromagnetic – spontaneous magnetization, large χ
- ...

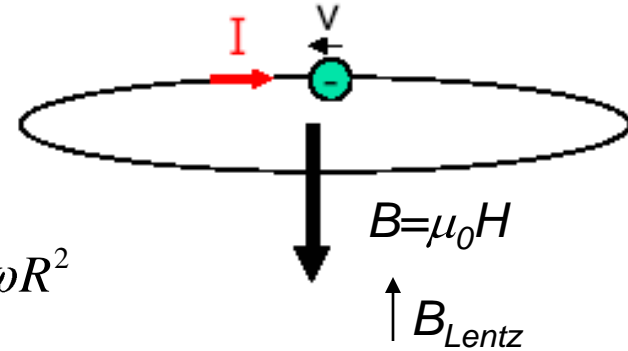
Material	χ_M (dim'less)
Bi	-1.6×10^{-4}
Ag	-2.4×10^{-5}
H ₂ O	-9.0×10^{-6}
Nb (4K)	-1
Na	8.5×10^{-6}
Al	2.1×10^{-5}
Pt	2.8×10^{-4}
Gd	0.48
Fe (annealed)	20,000
NdFe ₁₄ B	~ 0



From Cusack, 1963

Diamagnetism (classical)

- Arises from Lenz's law: when magnetic flux changes in a circuit, a current is induced which opposes the change of flux



- Orbiting electron creates magnetic dipole (circulating current)

$$I = \frac{\text{charge}}{\text{period}} = q \frac{\omega}{2\pi} \quad \mu = IA = \frac{1}{2} q\omega R^2$$

- In magnetic field, Lorentz's force is added to centrifugal force

$$m\omega^2 R = F - q\omega R \mu_0 H$$

- And corresponding change of rotational frequency

$$\omega^2 - \omega_0^2 = -\frac{q\omega\mu_0 H}{m}$$

- If change in orbital motion is small ($\omega \approx \omega_0$)

$$\Delta\omega = -\frac{q\mu_0 H}{2m}$$

- The energy associated with this frequency is

$$\Delta E = \hbar\Delta\omega = -\frac{q\hbar\mu_0 H}{2m} = -\mu_B \mu_0 H$$

Bohr magneton

$$\mu_B = \frac{q\hbar}{2m} = 9.274 \cdot 10^{-24} \left\{ \frac{\text{J}}{\text{T}} \right\} \equiv \text{A} \cdot \text{m}^2$$

- The change in frequency can be associated with induced magnetic dipole moment:

$$\mu_m = -\frac{q^2 R^2}{4m} \mu_0 H$$

Diamagnetism (classical) contd.

- Small magnetic field-induced magnetic dipole moment:

$$\mu_m = -\frac{q^2 R^2}{4m} \mu_0 H$$

- Now we can apply the result to spherical closed-shell atom

- Averaging over 3D gives mean square radial distance
- Sum over all Z electrons in the atom
- Sum over all atoms in a unit volume, density N , to obtain magnetization

$$\langle R^2 \rangle = \frac{3}{2} R^2 \begin{cases} \langle x^2 \rangle + \langle y^2 \rangle = R^2 \\ \langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle = \frac{1}{3} \langle R^2 \rangle \end{cases}$$

$$M = -\frac{q^2 ZN}{6m} \mu_0 H \langle R^2 \rangle$$

- Finally susceptibility

$$\chi = \frac{M}{H} = -\frac{q^2 ZN \mu_0}{6m} \langle R^2 \rangle$$

Larmor or Langevin diamagnetic susceptibility

- All atoms and ions display diamagnetic response
- Almost independent of temperature

Molar susceptibilities of some atoms and ions ($\times 10^{-6} \text{ cm}^3/\text{mole}$)

- Molar susceptibility is often used to describe magnetism of atoms (should be multiplied by molar volume to obtain dimensionless susceptibility)

<u>Ion</u>	χ_M	<u>Atom</u>	χ_M	<u>Ion</u>	χ_M
		He	-1.9	Li ⁺	-0.7
F ⁻	-9.4	Ne	-7.2	Na ⁺	-6.1
Cl ⁻	-24.2	A	-19.4	K ⁺	-14.6
Br ⁻	-34.5	Kr	-28	Rb ⁺	-22.0
I ⁻	-50.6	Xe	-43	Cs ⁺	-35.1

Paramagnetism

- Contrary to diamagnetism, paramagnetism arises from non-zero magnetic moments:
 - Free electron (Pauli) spin paramagnetism
 - Langevin atomic paramagnetism
- An electron has an intrinsic magnetic dipole moment associated with its spin S , equal to Bohr magneton:

$$\vec{\mu} = -g_s \frac{q}{2m} \vec{S}$$

$= 2.0023$
 $= \frac{\hbar}{2}$

$$\mu_B = \frac{q\hbar}{2m} = 9.274 \cdot 10^{-24} \left\{ \frac{\text{J}}{\text{T}} \right\} \equiv \text{A} \cdot \text{m}^2$$

- We can expect that the magnetic dipoles will rotate towards low-energy state ($U = \text{from } -\vec{\mu} \cdot \vec{B} \text{ to } +\vec{\mu} \cdot \vec{B}$)

For $B = 1 \text{ T}$ ($H = 8 \times 10^5 \text{ A/m}$)

$$U = \mu_B B = 58 \mu\text{eV} \rightarrow 0.67 \text{ K}$$

Field alignment is weak !

- The fraction of electrons with magnetic moments parallel to magnetic field exceeds the anti-parallel fraction by

$$\approx \frac{\mu_B \mu_0 H}{k_B T}$$

- For n free electrons, the magnetization
- But we need to take band structure into account !

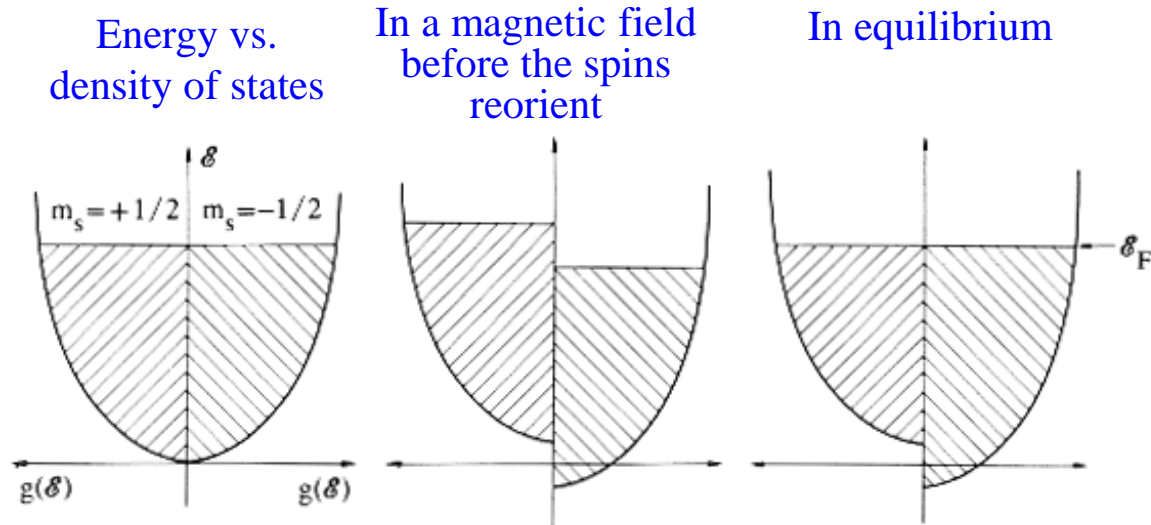
$$M \approx n \mu_B \frac{\mu_B \mu_0 H}{k_B T}$$

Paramagnetism of free spins

- Magnetization $M \approx n\mu_B \frac{\mu_B\mu_0 H}{k_B T}$
is ~100 times higher than observed in real materials

- In a band only a “thermal” fraction of electrons contributes to paramagnetism (compare to transport)

$$\approx \frac{k_B T}{E_F}$$



- Magnetization is $M \approx n\mu_B \frac{\mu_B\mu_0}{k_B T_F} H$

- Similar to transport, more accurate averaging over the distribution function gives susceptibility

$$\chi = \frac{3\mu_0}{2} \frac{n\mu_B^2}{k_B T_F}$$

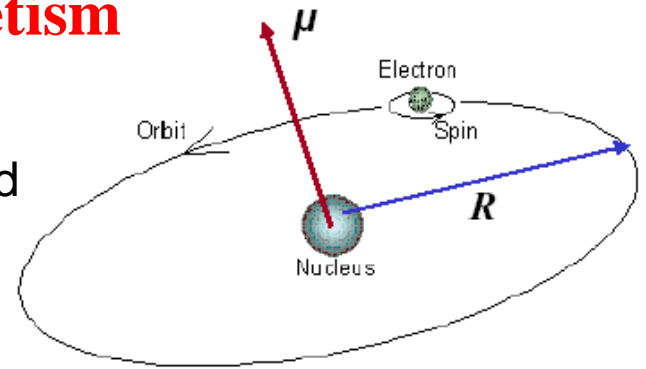
- For example, for Na $\chi = -8.4 \cdot 10^{-6}$

From Burns, 1990

Langevin atomic paramagnetism

- Similar to free spins, if an atom has a magnetic moment μ_{eff} , it can align along the magnetic field
- Magnetization of a material with atomic density N is (averaging included)

$$M = \frac{1}{3} N \mu_{eff} \frac{\mu_{eff} \mu_0 H}{k_B T}$$



- And susceptibility

$$\chi = \frac{\mu_0}{3} \frac{N \mu_{eff}^2}{k_B T}$$

$$\chi = \frac{C}{T}$$

Curie law for paramagnetics
With Curie constant

- Atom with orbital, spin and total angular momenta, L, S , and $J = L + S$, will have magnetic moment

$$\mu_{eff} = g_J \mu_B J$$

$$[= \mu_B \langle L + 2S \rangle]$$

$$C = \frac{N \mu_0 \mu_{eff}^2}{3k_B}$$

$$= 2.0023$$

With Landé g-factor

$$g_J = 1 + \frac{J^2 + S^2 - L^2}{2J^2}$$

- Complications

- Quantum mechanical averaging of m_J
- Ions
- Quenching of orbital momentum in the crystal field (Stark splitting of $2L+1$ degeneracy)

Atomic paramagnetism - Quantum theory

- Quantum mechanical averaging over $(2J+1)$ projections

$$M = N \frac{\sum_{-J}^{+J} g_J \mu_B m_J e^{\frac{g_J \mu_B m_J B}{k_B T}}}{\sum_{-J}^{+J} e^{\frac{g_J \mu_B m_J B}{k_B T}}} \equiv N g_J \mu_B J B_J(y) \equiv N \mu_{eff} B_J(y)$$

M_{sat} (pointing to $N \mu_{eff}$)

- With Brillouin function:

$$B_J(y) = \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J} y\right) - \frac{1}{2J} \coth\left(\frac{y}{2J}\right)$$

$$\text{with } y = \frac{\mu_{eff} B}{k_B T} = \frac{g_J \mu_B J B}{k_B T}$$

- If magnetic energy is small compared to thermal energy, $y \ll 1$, Brillouin function gives

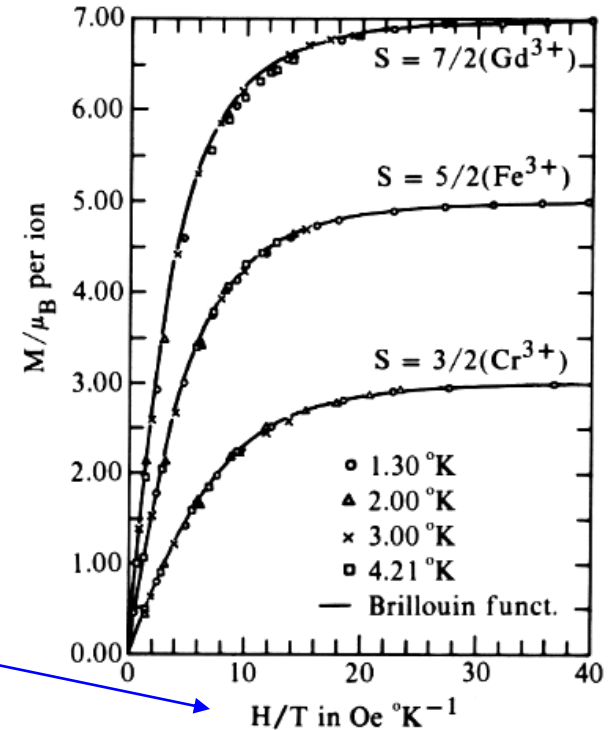
$$B_J(y \ll 1) \approx y \frac{J+1}{3J}$$

- This results in classic susceptibility

$$\chi = \frac{\mu_0}{3} \frac{N \mu_{eff}^2}{k_B T}$$

with quantum averaged $\mu_{eff}^2 = g_J \mu_B^2 J(J+1)$

Magnetic moment vs. H/T



Magnetic moments of ions

Ground states of ions predicted by Hund's rules

Values of magnetic moments of 4f and 3d ions in insulating compounds

f-shell ($\ell = 3$)									
n	$m_\ell = 3, 2, 1, 0, -1, -2, -3$	S	$L = \Sigma m_\ell $	J	$2S+1$	L_J	g_J		
0		0	0	0	1	S_0	0	La ³⁺	
1	↓	1/2	3	5/2	2	$F_{5/2}$	6/7	Ce ³⁺	
2	↓ ↓	1	5	4	3	H_4	4/5	Pr ³⁺	
3	↓ ↓ ↓	3/2	6	9/2	4	$I_{9/2}$	8/11	Nd ³⁺	
4	↓ ↓ ↓ ↓	2	6	4	5	I_4	3/5	Pm ³⁺	
5	↓ ↓ ↓ ↓ ↓	5/2	5	5/2	6	$H_{5/2}$	2/7	Sm ³⁺	
6	↓ ↓ ↓ ↓ ↓ ↓	3	3	0	7	F_0	–	Eu ³⁺	
7	↓ ↓ ↓ ↓ ↓ ↓ ↓	7/2	0	7/2	8	$S_{7/2}$	2	Gd ³⁺	
8	↑ ↑ ↑ ↑ ↑ ↑ ↑	3	3	6	9	F_6	3/2	Tb ³⁺	
9	↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑	5/2	5	15/2	10	$H_{15/2}$	4/3	Dg ³⁺	
10	↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑	2	6	8	11	I_8	5/4	Ho ³⁺	
11	↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑	3/2	6	16/2	12	$I_{15/2}$	6/5	Er ³⁺	
12	↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑	1	5	6	13	H_6	7/6	Tm ³⁺	
13	↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑	1/2	3	7/2	14	$F_{7/2}$	8/7	Yb ³⁺	
14	↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑	0	0	0	15	S_0	0	Lu ³⁺	

Ion	4f ⁿ	State	μ_{eff}/μ_B (Calc.)	μ_{eff}/μ_B (Exp.)
La ³⁺	4f ⁰	1S_0	0	0
Ce ³⁺	4f ¹	$^2F_{5/2}$	2.54	2.4
Pr ³⁺	4f ²	3H_4	3.58	3.5
Nd ³⁺	4f ³	$^4I_{9/2}$	3.62	3.5
Pm ³⁺	4f ⁴	5I_4	2.68	–
Sm ³⁺	4f ⁵	$^6H_{5/2}$	0.84	1.5
Eu ³⁺	4f ⁶	7F_0	0	3.4
Gd ³⁺	4f ⁷	$^8S_{7/2}$	7.94	8.0
Tb ³⁺	4f ⁸	7F_6	9.72	9.5
Dy ³⁺	4f ⁹	$^6H_{15/2}$	10.63	10.6
Ho ³⁺	4f ¹⁰	5I_8	10.60	10.4
Er ³⁺	4f ¹¹	$^4I_{15/2}$	9.59	9.5
Tm ³⁺	4f ¹²	3H_6	7.57	7.3
Yb ³⁺	4f ¹³	$^2F_{7/2}$	4.54	4.5

d-shell ($\ell = 2$)									
n	$m_\ell = 2, 1, 0, -1, -2$	S	$L = \Sigma m_\ell $	J					
1	↓	1/2	2	3/2	2	$D_{3/2}$	Ti ³⁺		
2	↓ ↓	1	3	2	3	F_2	V ³⁺		
3	↓ ↓ ↓	3/2	3	3/2	4	$F_{3/2}$	Cr ³⁺		
4	↓ ↓ ↓ ↓	2	2	0	5	D_0	Cr ²⁺		
5	↓ ↓ ↓ ↓ ↓	5/2	0	5/2	6	$S_{5/2}$	Fe ³⁺ , Mn ²⁺		
6	↑ ↑ ↑ ↑ ↑	2	2	4	7	D_4	Fe ²⁺		
7	↑ ↑ ↑ ↑ ↑ ↑	3/2	3	9/2	8	$F_{9/2}$	Co ²⁺		
8	↑ ↑ ↑ ↑ ↑ ↑ ↑	1	3	4	9	F_4	Ni ²⁺		
9	↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑	1/2	2	5/2	10	$D_{5/2}$	Cu ²⁺		
10	↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑	0	0	0	11	S_0			

Ion	3d ⁿ	State	μ_{eff}/μ_B (Calc.-J)	μ_{eff}/μ_B (Calc.-J=S)	μ_{eff}/μ_B (Exp.)
Ti ³⁺ , V ⁴⁺	3d ¹	$^2D_{3/2}$	1.55	1.73	1.8
V ³⁺	3d ²	3F_2	1.63	2.83	2.8
Cr ³⁺ , V ²⁺	3d ³	$^4F_{3/2}$	0.77	3.87	3.8
Mn ³⁺ , Cr ²⁺	3d ⁴	5D_0	0	4.90	4.9
Fe ³⁺ , Mn ²⁺	3d ⁵	$^6S_{5/2}$	5.92	5.92	5.9
Fe ²⁺	3d ⁶	5D_4	6.70	4.90	5.4
Co ²⁺	3d ⁷	$^4F_{9/2}$	6.63	3.87	4.8
Ni ²⁺	3d ⁸	3F_4	5.59	2.83	3.2
Cu ²⁺	3d ⁹	$^2D_{5/2}$	3.55	1.73	1.9

Ferromagnetism – Molecular field theory

- Spontaneous magnetization occurs in some (Ferromagnetic) materials composed of atoms with unfilled shells
- For some reason magnetic moments are aligned even at relatively high temperature
- Hypothesis: magnetic order is due to strong local magnetic field (Weiss effective field) at the site of each dipole

$$B_{loc} = B_a + \mu_0 \gamma M$$

with a constant γ

- Consider a collection of N identical atoms per unit volume, with total angular momentum J , and use QM treatment of atomic paramagnetism

$$M = M_{sat} B_J \quad y \quad M_{sat} = N \mu_{eff}$$

$$\text{with } y = \frac{\mu_{eff} B_{loc}}{k_B T} \equiv \frac{\mu_{eff} B_a + \mu_0 \gamma M}{k_B T} = \frac{g_J \mu_B J}{k_B T} \frac{B_a + \mu_0 \gamma M}{k_B T}$$

Values of Curie temperatures and spontaneous magnetism (at 0 K in Gauss) for a few ferromagnetic materials

(a)	<u>Mat.</u>	$T_c(^{\circ}\text{K})$	M_s	<u>Mat.</u>	$T_c(^{\circ}\text{K})$	M_s
	Fe	1043	1752	CrBr ₃	37	270
	Co	1388	1446	Au ₂ MnAl	200	323
	Ni	627	510	Cu ₂ MnAl	630	726
	Gd	293	1980	Cu ₂ MnIn	500	613
	Dy	85	3000	MnAs	318	870
	EuO	77	1910	MnBi	670	675
	EuS	16.5	1184	GdCl ₃	2.2	550

(b)	<u>Mat.</u>	$T_N(^{\circ}\text{K})$	<u>Mat.</u>	$T_N(^{\circ}\text{K})$
	MnO	122	KCoF ₃	125
	FeO	198	MnF ₂	67.34
	CoO	291	FeF ₂	78.4
	NiO	600	CoF ₂	37.7
	RbMnF ₃	54.5	MnCl ₂	2
	KFeF ₃	115	VS	1040
	KMnF ₃	88.3	Cr	311

(c)	<u>Mat.</u>	$T_c(^{\circ}\text{K})$	M_s	<u>Mat.</u>	$T_c(^{\circ}\text{K})$	M_s
	Fe ₃ O ₄	858	510	CuFe ₂ O ₄	728	160
	CoFe ₂ O ₄	793	475	MnFe ₂ O ₄	573	560
	NiFe ₂ O ₄	858	300	Y ₃ Fe ₅ O ₁₂	560	195

From Burns, 1990

Ferromagnetism – Molecular field theory

- Now let's find the spontaneous magnetization ($B_a = 0$)

$$M = M_{sat} B_J y \quad \text{with} \quad y = \frac{\mu_{eff} \mu_0 \gamma M}{k_B T}$$

Solving equation against y :

$$y \frac{k_B T}{\mu_{eff} \mu_0 \gamma M_{sat}} = B_J y$$

- Depending on temperature spontaneous magnetization can be either $M=0$ or finite
- At low y $B_J (y \ll 1) \approx y \frac{J+1}{3J}$

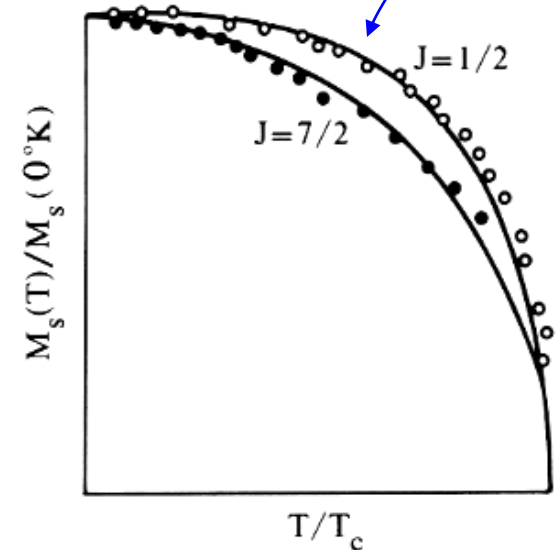
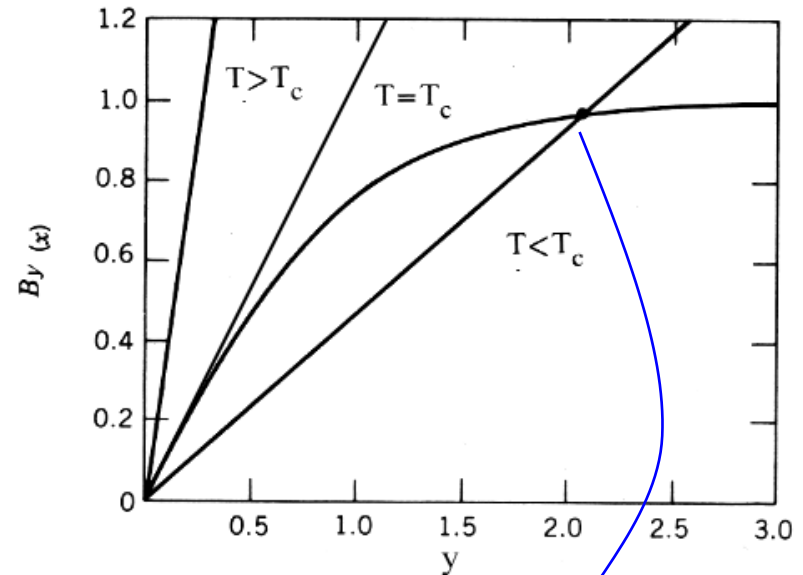
We can find critical Curie temperature:

$$\frac{k_B T_C}{\mu_{eff} \mu_0 \gamma M_{sat}} = \frac{J+1}{3J}$$

$$T_C = \frac{N \mu_0 \gamma}{3 k_B} \overbrace{g_J \mu_B^2 J J + 1}^{\mu_{eff}^2}$$

From Burns, 1990

Solution of equation with Brillouin function



Ferromagnetism – Molecular field theory

- At temperatures $T > T_c$, there is no spontaneous magnetization, and we can find temperature dependence of magnetization

($B_a > 0$)

$$M = M_{sat} B_J y$$

$$\text{with } y = \frac{g_J \mu_B J}{k_B T} (B_a + \mu_0 \gamma M)$$

$$M = M_{sat} y \frac{J+1}{3J} = \frac{N \mu_{eff}^2}{3k_B T} (B_a + \mu_0 \gamma M)$$

- Solving for M :

$$M = \frac{C}{\mu_0 (T - C\gamma)} B_a$$

with Curie constant

$$C = \frac{N \mu_0 \mu_{eff}^2}{3k_B} = \gamma T_c$$

- Then susceptibility of ferromagnet in paramagnetic region

$$\chi = \frac{C}{T - T_c}$$

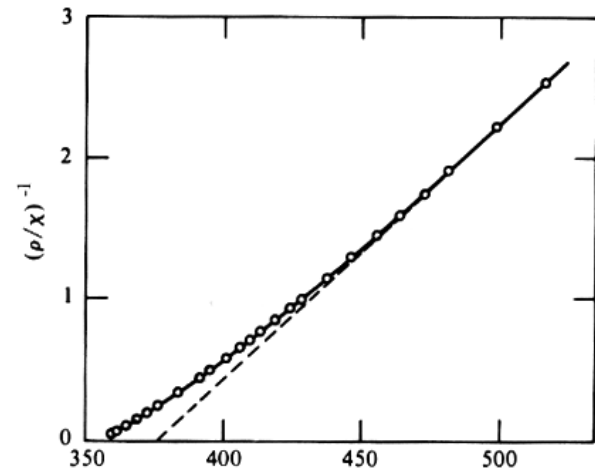
- Some estimation for iron:

$$g_J = 2; \quad J = 1; \quad N = 8.5 \cdot 10^{28} m^{-3} \Rightarrow C = 1.77 K$$

$$T_c = 1043 K; \quad \Rightarrow \gamma = 588$$

$$M \approx 1700 \text{ Gauss}; \quad \Rightarrow B \sim 10^6 \text{ Gauss} = 100 T$$

Reciprocal susceptibility vs. temperature for nickel



Temperature (C) From Burns, 1990

NNSE 508 EM Lecture #12

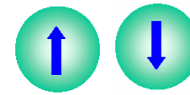
Huge !

Ferromagnetism – Heisenberg exchange interaction

- What is the reason for so high local magnetic field ? – Exchange interaction
- Consider two electrons on two atoms. We need to find the energy difference = exchange integral J_{ex}) :
- Their wavefunction is antisymmetric due to Pauli exclusion principle
- Usually the interaction between the **space and spin** parts is small, and the variables can be separated :
- Antiparallel spins give antisymmetric spin wavefunction, etc.
- We can construct wavefunctions for singlet and triplet states with correct symmetry:

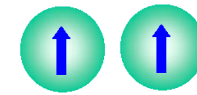
$$H_{ex} = -J_{ex} \mathbf{s}_1 \cdot \mathbf{s}_2$$

Singlet



$$\mathbf{s}_1 \cdot \mathbf{s}_2 = -\frac{3}{4}$$

Triplet



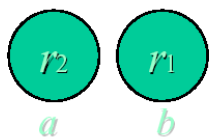
$$\mathbf{s}_1 \cdot \mathbf{s}_2 = +\frac{1}{4}$$

$$\Psi(r_1, s_1, r_2, s_2) = -\Psi(r_2, s_2, r_1, s_1)$$

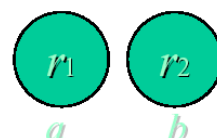
$$\Psi(r_1, s_1, r_2, s_2) = \varphi_{\text{space}}(r_1, r_2) g_{\text{spin}}(s_1, s_2)$$

Wavefunction	Singlet	Triplet
Total	Antisym.	Antisym.
Spin part	Antisym.	Symmetric
Space part	Symmetric	Antisym.

$$\Psi_1(\mathbf{r}_1, \mathbf{r}_2) = \phi_a(\mathbf{r}_1)\phi_b(\mathbf{r}_2)$$



$$\Psi_2(\mathbf{r}_1, \mathbf{r}_2) = \phi_a(\mathbf{r}_2)\phi_b(\mathbf{r}_1)$$



$$\varphi_S(x_1, x_2) = \phi_a(r_1)\phi_b(r_2) + \phi_a(r_2)\phi_b(r_1) = \Psi_1 + \Psi_2$$

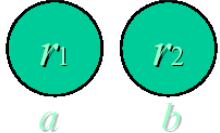
$$\varphi_T(x_1, x_2) = \phi_a(r_1)\phi_b(r_2) - \phi_a(r_2)\phi_b(r_1) = \Psi_1 - \Psi_2$$

Ferromagnetism – Heisenberg exchange interaction-contd

- The energy shift of singlet and triplet states can be calculated from perturbation theory:

$$E_S = \langle \phi_S | V | \phi_S \rangle = \langle \Psi_1 | V | \Psi_1 \rangle + \langle \Psi_2 | V | \Psi_2 \rangle + \langle \Psi_1 | V | \Psi_2 \rangle + \langle \Psi_2 | V | \Psi_1 \rangle$$

$$E_T = \langle \phi_T | V | \phi_T \rangle = \langle \Psi_1 | V | \Psi_1 \rangle + \langle \Psi_2 | V | \Psi_2 \rangle - \langle \Psi_1 | V | \Psi_2 \rangle - \langle \Psi_2 | V | \Psi_1 \rangle$$

$$V_{r_1, r_2} = \frac{e^2}{\epsilon_0} \left[\frac{1}{r_{ab}} - \frac{1}{r_{a2}} - \frac{1}{r_{b1}} + \frac{1}{r_{12}} \right]$$


- The energy difference between the singlet and triplet states

$$E_S - E_T = J_{exc} = 4 \langle \Psi_1 | V | \Psi_2 \rangle = 4 \langle \phi_a r_1 \phi_b r_2 | V | \phi_a r_2 \phi_b r_1 \rangle$$

$$= \frac{e^2}{\epsilon_0} \left[-8 \left\langle \phi_a r_1 \phi_b r_2 \left| \frac{1}{r_{a2}} \right| \phi_a r_2 \phi_b r_1 \right\rangle + 4 \left\langle \phi_a r_1 \phi_b r_2 \left| \frac{1}{r_{ab}} \right| \phi_a r_2 \phi_b r_1 \right\rangle + 4 \left\langle \phi_a r_1 \phi_b r_2 \left| \frac{1}{r_{12}} \right| \phi_a r_2 \phi_b r_1 \right\rangle \right]$$

- At small a - b distance $J_{exc} < 0$ and singlet state is favorable
- At large a - b distance $J_{exc} > 0$ and triplet state is favorable
- BTW if the electrons are on the same atom, the ion interaction change is zero, $J_{exc} > 0$ and antiparallel spins are favorable = Hund's rule

Ferromagnetism – Heisenberg exchange interaction contd.

- Exchange interaction can be ferromagnetic or antiferromagnetic depending on interatomic distance
- Exchange interaction is electrostatic (strong) in nature
- To correlate it with molecular field theory, we can write:

$$H_{ex} = - \sum_{i,j} J_{ex} S_i \cdot S_j \equiv \sum_i g_J \mu_B S_i B_{loc}$$

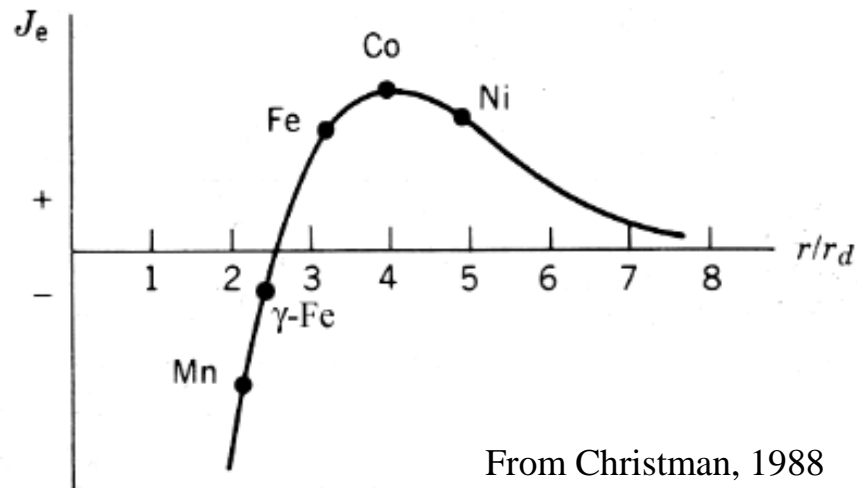
$$z J_{ex} S = g_J \mu_B B_{loc}$$

↖ Number of nearest neighbors

- For Fe: $J_{ex} = \frac{g_J \mu_B \mu_0 \gamma M}{z S} = \frac{g_J \mu_B^2 \mu_0 \gamma n}{z} \approx 11 \text{ meV}$
- Electrostatic interaction easily accounts for this value

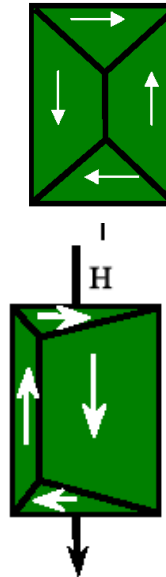
Exchange integral vs. interatomic distance

r_d – average radius of 4d electron

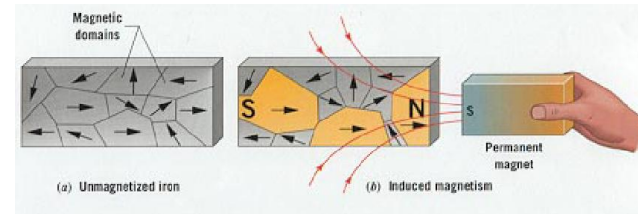


Ferromagnetic materials

- Energy is minimized by ordering spins into “domains”
 - Net moment, \mathbf{M} , would cause external field, increase energy
 - Magnetic domains cancel so that $\mathbf{M} = 0$
 - Natural ferromagnetism does not produce net magnetic field
- To magnetize a ferromagnet, impose \mathbf{H}
 - Domain walls move to align \mathbf{M} and \mathbf{H}
 - Defects impede domain wall motion
 - Magnetization (\mathbf{M}_r) retained when \mathbf{H} removed



Ferromagnetic material is always locally saturated



- Magnetic properties

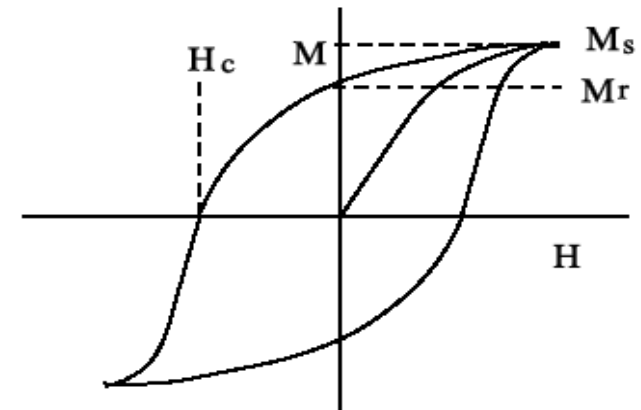
M_s = saturation magnetization (All spins aligned with field)

M_r = remanent magnetization (Useful moment of permanent magnet)

H_c = coercive force (Field required to “erase” moment)

Area inside curve = magnetic hysteresis (Governs energy lost in magnetic cycle)

Hysteresis loop of a ferromagnetic material



Core magnetism – materials with spontaneously ordered magnetic dipoles

- High temperature:
 - Spins disordered \Rightarrow paramagnetism
- Low Temperature ($T < T_c$)
 - Spins align = ferromagnetism
 - Elements: Fe, Ni, Co, Gd, Dy
 - Alloys and compounds: AlNiCo, FeCrCo, SmCo₅, Fe₁₄Nd₂B
 - Like spins alternate = antiferromagnetism (RbMnF₃)
 - Unlike spins alternate = ferrimagnetism
 - Compounds: Fe₃O₄ (lodestone, magnetite), CrO₃, SrFe₂O₃, other ferrites and garnets



Ferromagnetism

$$\mathbf{M} = n\mathbf{m}$$



antiferromagnetism

$$\mathbf{M} = 0$$



ferrimagnetism

$$\mathbf{M} = (n/2)(\mathbf{m}_1 - \mathbf{m}_2)$$

Other types of exchange interaction

- Superexchange (transition metal oxides)
 - Can be ferromagnetic or antiferromagnetic depending upon the energy of delocalization of the p-electrons on M_1 and M_2
 - Ordering temperature up to 900 K in ferrites (NiFe_2O_4 - 863 K)
 - Sign mostly negative, though ferromagnetics are known: EuO ($T_c=69\text{K}$) or CrBr_3 ($T_c=37\text{K}$)

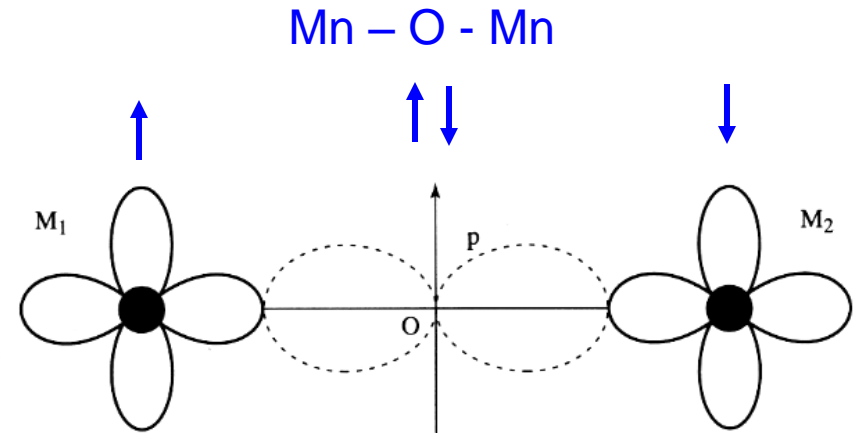
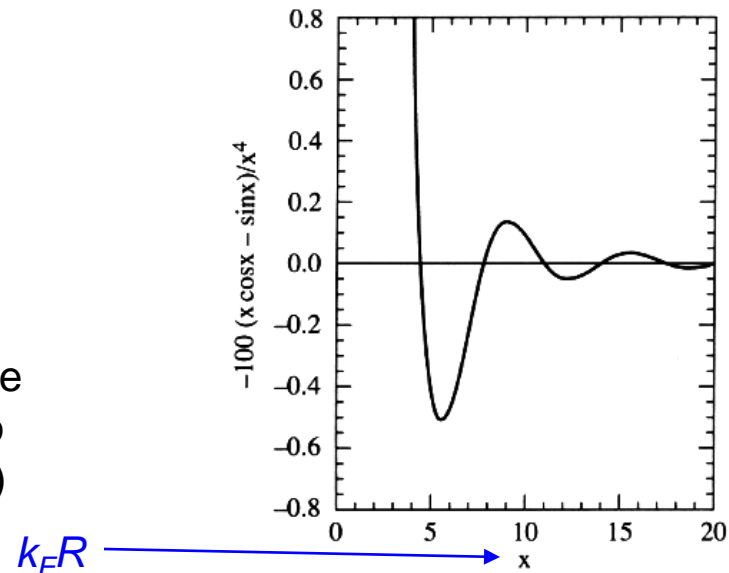


Figure 9.1 - M_1 and M_2 : transition metal ions, O : O^{2-}

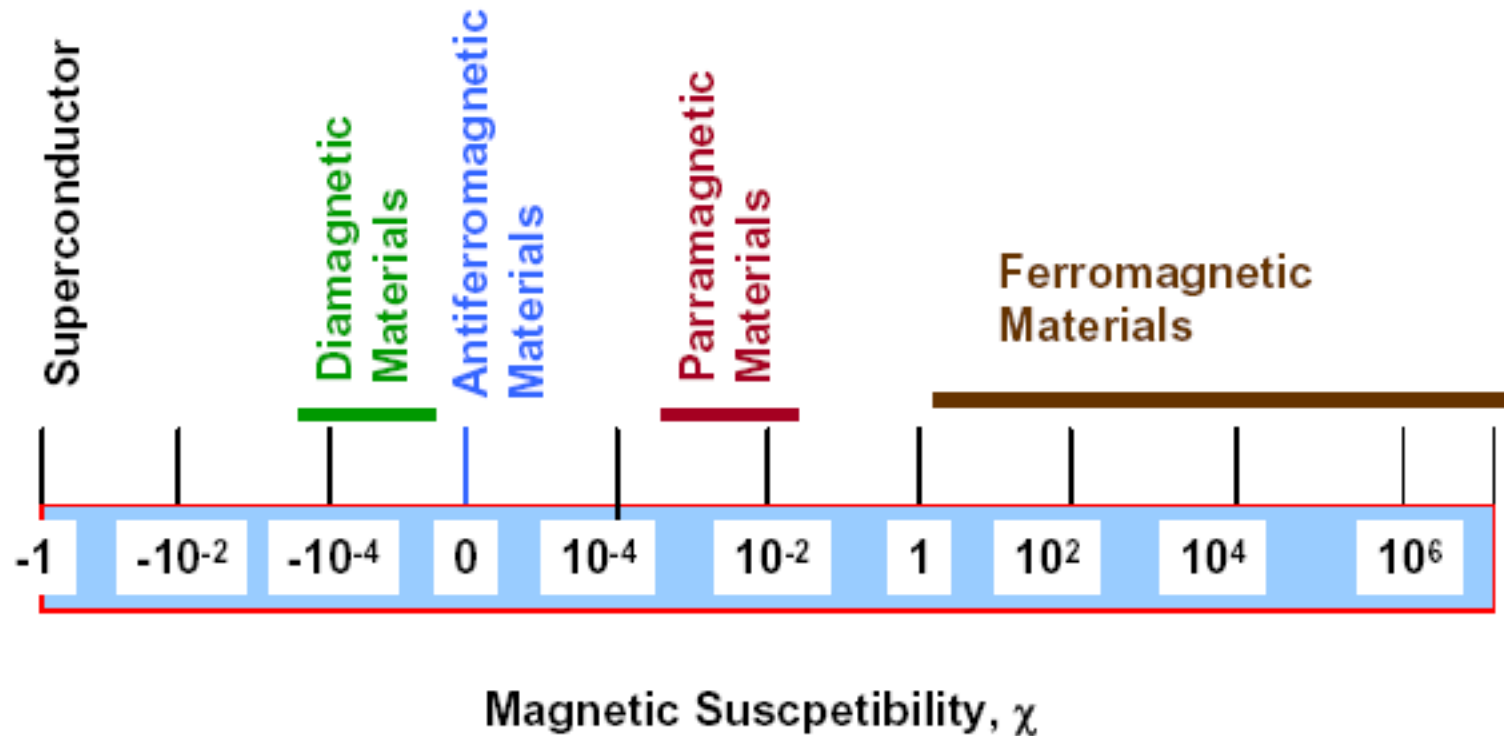
- RKKY interaction (Ruderman-Kittel-Kasuya-Yosida) - Indirect exchange over relatively large distances through spin of conduction electrons (4f metals)
 - Interaction oscillates with $(k_F R)$, Fermi wavevector determines the wavelength of oscillations
 - The interaction is of the same order for all rare earths, but ordering temperatures vary due to magnetic moment: 19K for Nd, 289 K for Gd)

RKKY interaction



Magnetic materials

- Magnetic induction field is the same in a



From Goldberg, 2006

NNSE 508 EM Lecture #12

Magnetic units

Magnetic induction (tesla): $1 \text{ T} = 10^4 \text{ gauss} (= 1 \text{ Wb} \cdot \text{m}^{-2})$.

B can be called magnetic induction, magnetic induction field or magnetic flux density.

Magnetic field (ampere / metre): $1 \text{ A} \cdot \text{m}^{-1} = 4\pi \times 10^{-3} \text{ oersted}$.

The magnetic field **H** is often expressed in units of $\mu_0 H$, hence in tesla (T) or its submultiple, the gamma (γ) which is equal to 10^{-9} T . A field of $1 \text{ A} \cdot \text{m}^{-1}$ corresponds to $1.2566 \mu\text{T}$.

Magnetisation (ampere / metre): $1 \text{ A} \cdot \text{m}^{-1} = 10^{-3} \text{ emu} \cdot \text{cm}^{-3}$.

Magnetic moment (ampere-square metre or joule per tesla): $1 \text{ A} \cdot \text{m}^2 = 1 \text{ J} \cdot \text{T}^{-1} = 10^3 \text{ emu}$.

Specific magnetic moment: $1 \text{ A} \cdot \text{m}^2 \cdot \text{kg}^{-1} = 1 \text{ emu} \cdot \text{g}^{-1}$.

Note that, sometimes, magnetisation (**M**), magnetic moment (**m**) and specific magnetic moment (**σ**) are expressed in tesla, Weber-meter and Weber-meter per kilogram, respectively : the reason is that we have adopted as definition of magnetisation ($d\mathbf{m} / dV$): $\mathbf{M} = \mathbf{B} / \mu_0 - \mathbf{H}$, whereas some authors call “magnetisation” the quantity $\mathbf{B}_i = \mathbf{J} = \mathbf{B} - \mu_0 \mathbf{H}$ which is usually called “(magnetic) polarisation”.

$$1 \text{ Wb} = \text{T} \cdot \text{m}^2$$

$$1 \text{ T} = \frac{\text{V} \cdot \text{s}}{\text{m}^2}$$

$$1 \text{ H} = \frac{\text{Wb}}{\text{A}} = \frac{\text{T} \cdot \text{m}^2}{\text{A}}$$

From Tremolet de Lacheisserie, 2005

Physical constants

Name and symbol	Numerical value
Speed of light in vacuum, c	$2.9979 \times 10^8 \text{ m} \cdot \text{s}^{-1}$
Permeability of vacuum, μ_0	$4\pi \times 10^{-7} \text{ H} \cdot \text{m}^{-1}$
Permittivity of vacuum, $\epsilon_0 = 1/c^2\mu_0$	$8.8542 \times 10^{-12} \text{ F} \cdot \text{m}^{-1}$
Planck's constant, h	$6.6261 \times 10^{-34} \text{ J} \cdot \text{s}$
$\hbar = h/2\pi$	$1.0546 \times 10^{-34} \text{ J} \cdot \text{s}$
Acceleration of gravity, g	$9.8066 \text{ m} \cdot \text{s}^{-2}$
Electron rest mass, m	$9.1094 \times 10^{-31} \text{ kg}$
Electron charge (absolute value), e	$1.6022 \times 10^{-19} \text{ C}$
Bohr magneton, $\mu_B = e\hbar/2m$	$9.2742 \times 10^{-24} \text{ A} \cdot \text{m}^2$
Flux quantum, $h/2e$	$2.0678 \times 10^{-15} \text{ Wb}$
Avogadro's number N	$6.0221 \times 10^{23} \text{ mol}^{-1}$
Boltzmann's constant, k_B	$1.3807 \times 10^{-23} \text{ J} \cdot \text{K}^{-1}$

Length (metre)	: $1 \text{ m} = 10^2 \text{ cm} = 39.37 \text{ " (inch)} = 10^{10} \text{ angström (Å)}$.
Force (newton)	: $1 \text{ N} = 10^5 \text{ dyn} = 0.102 \text{ kgf}$.
Energy (joule)	: $1 \text{ J} = 10^7 \text{ erg} = 0.7243 \times 10^{23} \text{ K} = 0.6241 \times 10^{19} \text{ eV}$.
Energy density	: $1 \text{ J} \cdot \text{m}^{-3} = 10 \text{ erg} \cdot \text{cm}^{-3}$.
Power (watt)	: $1 \text{ W} = 10^7 \text{ erg} \cdot \text{s}^{-1} = 1.359 \times 10^{-3} \text{ CV} = 1.340 \times 10^{-3} \text{ hp}$.
Pressure (pascal)	: $1 \text{ Pa} = 10 \text{ baryes} = 10^{-5} \text{ bar} = 1.02 \times 10^{-5} \text{ kgf} \cdot \text{cm}^{-2}$ $= 7.49 \times 10^{-3} \text{ torrs} = 1.45 \times 10^{-4} \text{ psi}$.