Lecture contents

- **Magnetic field**
  - **Ampere's law**
  - **Lorentz force, cyclotron frequency,**
  - **Hall effect**
  - **Dipole moment, circulation electron, spin**

Magnetostatics: Ampere’s Law of Force

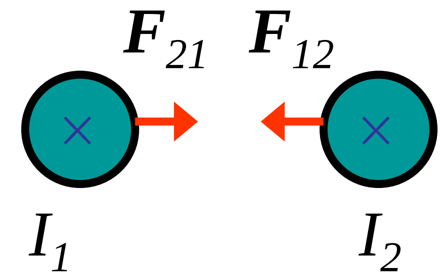
- *Ampere’s law of force* is the “law of action” between current carrying circuits through magnetic field.
- Experimental facts:
  - The magnitude of the force is inversely proportional to the distance squared.
  - The magnitude of the force is proportional to the product of the currents carried by the two wires.
- The force acting on a current element $I_2\, d\mathbf{l}_2$ by a current element $I_1\, d\mathbf{l}_1$ is given by

$$\delta \vec{F}_{12} = \frac{\mu_0}{4\pi}\frac{I_2\delta\vec{l}_2 \times \; I_1\delta\vec{l}_1 \times \vec{a}_{R_{12}}}{R_{12}^2}$$

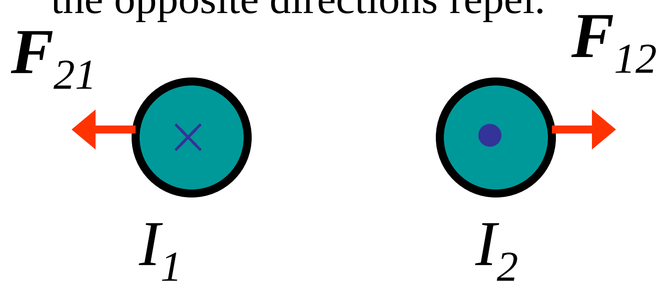
Permeability of free space
$\mu_0 = 4\pi \times 10^{-7}$ {FΩ/m} ={N/A^2}

unit vector in direction of I_2 from I_1

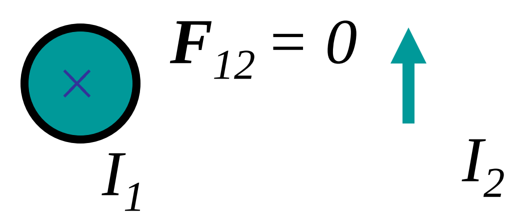
- Experimental facts:
  - Two parallel wires carrying current in the same direction attract.



  - Two parallel wires carrying current in the opposite directions repel.



  - A short current oriented perpendicular to another current experiences no force.



Magnetic field

$$\delta\vec{F}_{12} = \frac{\mu_0}{4\pi}\frac{I_2\delta\vec{l}_2 \times \; I_1\delta\vec{l}_1 \times \vec{a}_{R_{12}}}{R_{12}^2}$$

- The total force acting on a circuit C_2 having a current I_2 by a circuit C_1 having current I_1 is given by

$$\vec{F} = \frac{\mu_0 I_1 I_2}{4\pi}\oint_{C_2}\oint_{C_1}\frac{d\vec{l}_2 \times \; d\vec{l}_1 \times \vec{a}_{R_{12}}}{R_{12}^2}$$

$$\vec{F} = \oint_{C_2} I_2 d\vec{l}_2 \times \frac{\mu_0}{4\pi}\oint_{C_1}\frac{I_1 d\vec{l}_1 \times \vec{a}_{R_{12}}}{R_{12}^2}$$

- Similar to Coulomb's law, magnetic field can be introduced

$$\vec{B} = \frac{\mu_0}{4\pi}\oint_{C_1}\frac{I_1 d\vec{l}_1 \times \vec{a}_{R_{12}}}{R_{12}^2} \quad \mathrm{T}$$

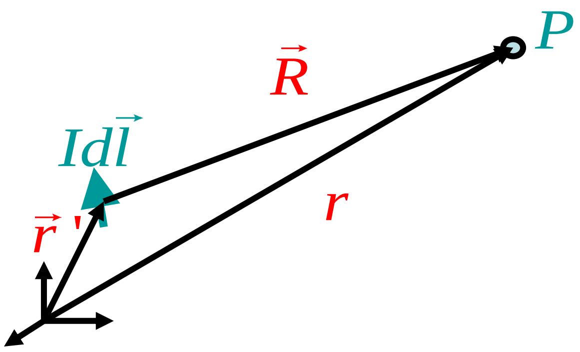
- An infinitely small current element $Id\boldsymbol{l}$ immersed in a region of magnetic flux density $\boldsymbol{B}$, experiences a force $d\boldsymbol{F}$

$$d\vec{F} = Id\vec{l} \times \vec{B}$$

The Biot-Savart Law

- The contribution to the *B*-field at a point *P* from a differential current element *Id**l*** is given by

$$d\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\vec{R}}{R^3}$$



- The total magnetic flux at the point *P* due to the entire circuit *C* is given by

$$\vec{B}(\vec{r}) = \oint_C \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\vec{R}}{R^3}$$

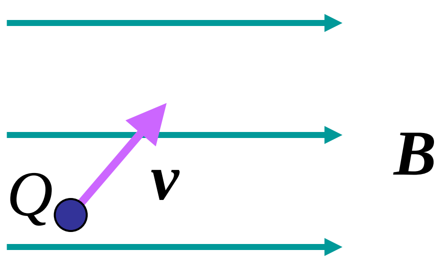
Lorentz Force

$$d\vec{F} = Id\vec{l} \times \vec{B}$$

$$Id\vec{l} = Q\vec{v}$$

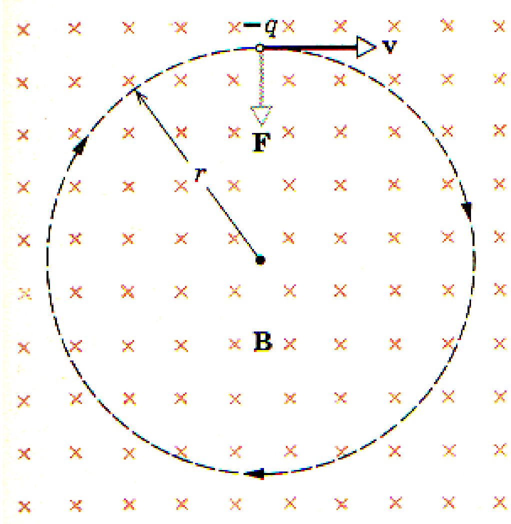
- A moving point charge placed in both electric and magnetic field experiences a force given by

$$\vec{F} = \vec{F}_e + \vec{F}_m = q \quad e + \vec{v} \times \vec{B}$$



The force experienced by the point charge is in the direction into the paper.

Cyclotron Frequency



- Electron circulating in a uniform magnetic field
- Magnetic field causes a centripetal force

$$\vec{F} = q\ \vec{v}\times\vec{B}\ = qvB = \frac{mv^2}{r} \qquad r = \frac{mv}{qB}$$

- Angular frequency does not depend on electron velocity

$$\omega_c = \frac{v}{r} = \frac{qB}{m} \qquad \nu_c = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$$

- Cyclotron resonance (CR) used for effective mass measurements, in particle accelerators (cyclotrons, synchrotrons).

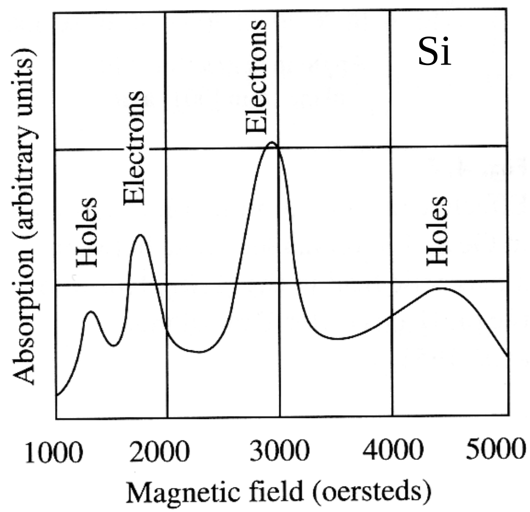


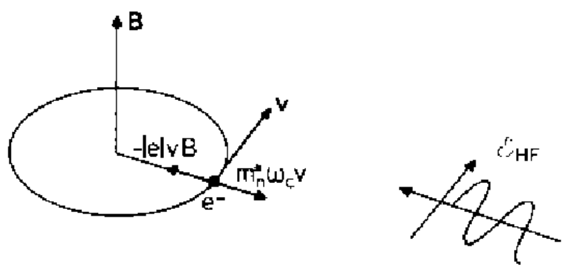
Fig. 4.6
Cyclotron resonance absorption versus magnetic field at 24000 Mc/s for Si at 4 K (after Dresselhaus *et al.* 1955).

Set-up:

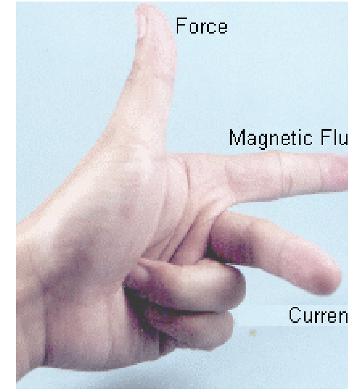
- Place crystal in static magnetic field ***B***
- measure absorption of RF electric field ***E***
- keep ***E*** constant and change ***B***: $\mu(B)$

Large mean free path of carriers (long scattering times) is needed for CR measurements :

$$\tau\omega_c > 1$$



Hall effect: carrier charge and concentration determination



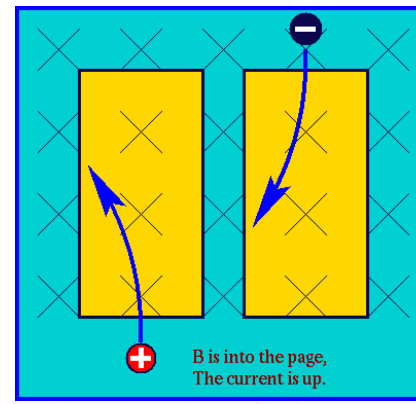
Lorentz force on a moving particle:

$$F = e(v \times B)$$

Steady state: balance of forces in y direction:

$$eE_y = ev_x B_z$$

<u>Consider one type of carriers (e.g. *n* >> *p*)</u>

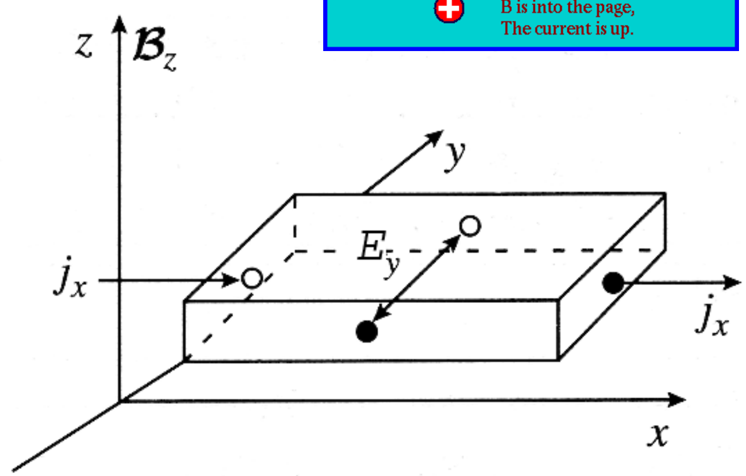


Current density in x-direction:

$$J_x = env_x$$

Hall coefficient (Hall and drift mobilities considered equal):

$$R_H = \frac{E_y}{J_x B_z} = \begin{cases} -\frac{1}{ne} & for \quad n-type \\ \frac{1}{pe} & for \quad p-type \end{cases}$$



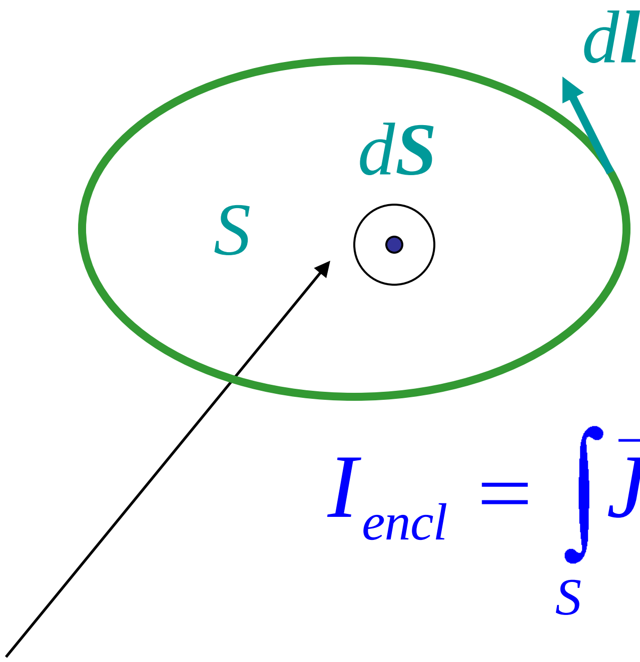
Ampere's Circuital Law in Integral Form

- ***Ampere's Circuital Law*** in integral form states that the circulation of the magnetic field is proportional to the total current through the surface bounding the path over which the circulation is computed.

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- Just as Gauss's law follows from Coulomb's law, so Ampere's circuital law follows from Ampere's force law.

- Just as Gauss's law can be used to derive the electrostatic field from symmetric charge distributions, so Ampere's law can be used to derive the magnetostatic field from symmetric current distributions.



$$I_{encl} = \int_S \vec{J} \cdot d\vec{s}$$

By convention, ***dS*** is taken to be in the direction defined by the right-hand rule applied to ***dl***.

Since volume current density is the most general, we can write I_{encl} in this way.

Applications of Ampere's Law

- Ampere's law in integral form is an *integral equation* for the unknown magnetic flux density resulting from a given current distribution.

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

known (I_{encl}), unknown ($\vec{B}$)

- For example, magnetic field around a straight wire with a current I at a distance r:

$$B \cdot 2\pi r = \mu_0 I \qquad B = \frac{\mu_0 I}{2\pi r}$$

- Another example: field inside a solenoid is determined by a number of turns per length *n* and current *I* (no core):

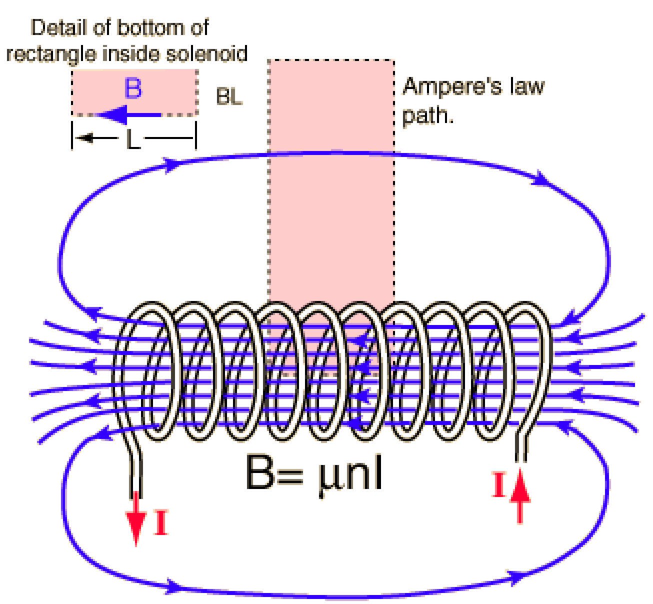
$$BL = \mu_0 NI$$

$$\boxed{B = \mu_0 nI}$$

Field outside is small

- If core with relative permittivity μ is inserted

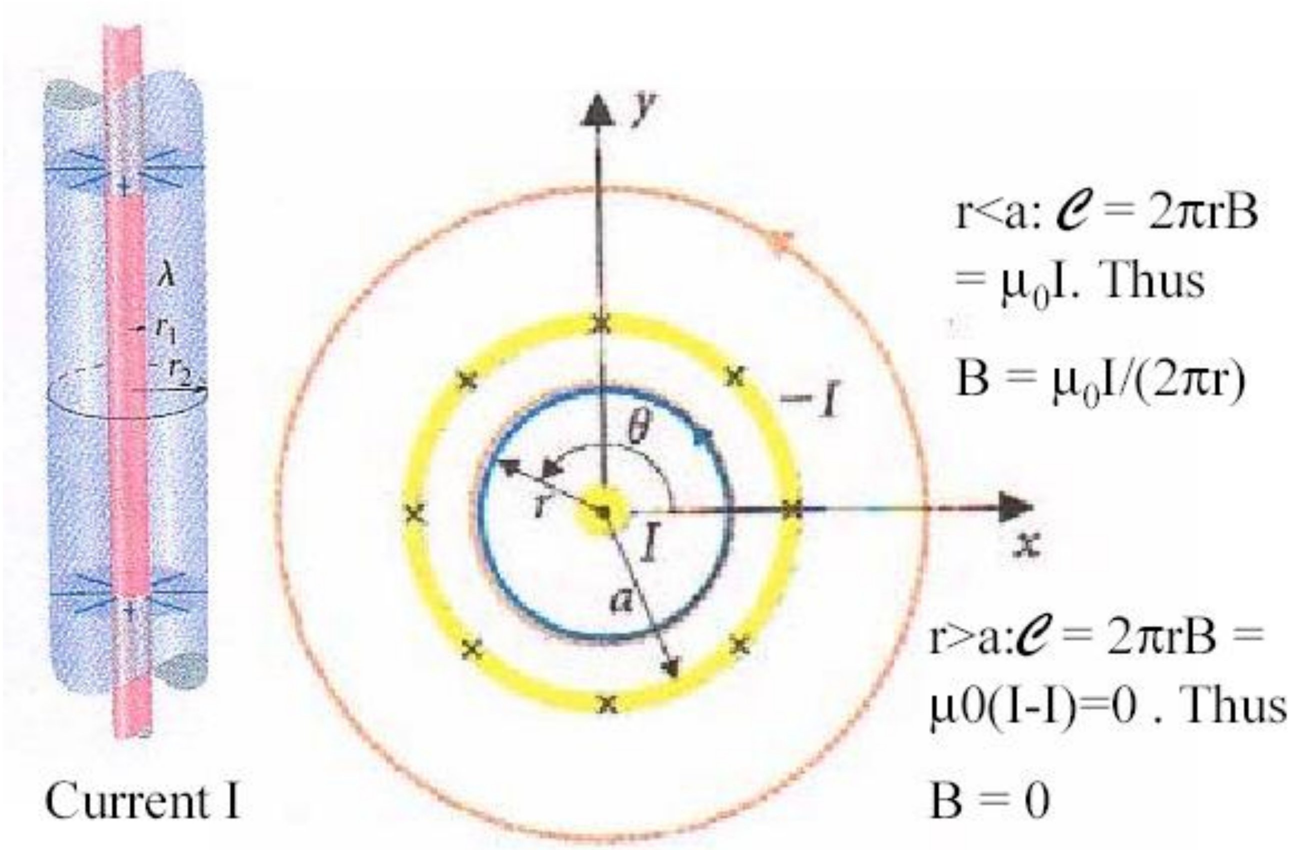
$$B = \mu_0 \mu nI$$



Applications of Ampere's Law: Coaxial cable

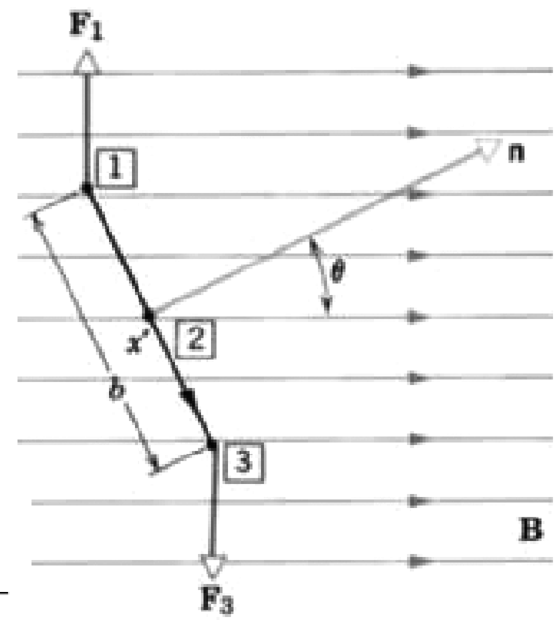
$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- Coaxial cable: thin internal wire carriers current I and coaxial metal cylinder carries $-I$
- Field is confined inside the outer cylinder



Magnetic dipole moment

- A ***magnetic dipole*** comprises a small current carrying loop.
- The point charge (*charge monopole*) is the simplest source of electrostatic field. The magnetic dipole is the simplest source of magnetostatic field. The magnetic dipole is analogous to the electric dipole.
- Torque on a current loop in a uniform filed:



$$d\vec{F} = I d\vec{l} \times \vec{B}$$

$$\begin{aligned}\vec{\tau} &= \sum_i \vec{r}_i \times \vec{F}_i \\ &= (b/2)(-\sin\theta\hat{i} + \cos\theta\hat{j}) \times IaB\hat{j} + \\ &+ (b/2)(\sin\theta\hat{i} - \cos\theta\hat{j}) \times IaB(-\hat{j}) \\ &= (b/2)IaB(-\sin\theta\hat{k} - \sin\theta\hat{k}) \\ &= -IabB\sin\theta\hat{k}\end{aligned}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\vec{\mu} = IA\vec{n} \qquad \mathrm{Am}^2$$

- The magnetic dipole moment can be defined as current times the area of the loop
- Similar to an electric dipole: $\vec{\tau} = \vec{\mu} \times \vec{B}$

Direction of the dipole moment is determined by the direction of current using the right-hand rule .

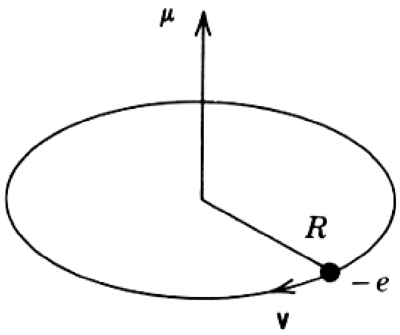
$$\vec{B} = \frac{\mu_0}{4\pi}\left[\frac{3\ \vec{\mu}\bullet\vec{r}\ \ \vec{r}}{r^5} - \frac{\vec{\mu}}{r^3}\right]$$

$$U = from \quad -\vec{\mu}\cdot\vec{B} \quad to \quad +\vec{\mu}\cdot\vec{B}$$

Circulating electron and spin

- Electron circulating with uniform angular speed ω produces current:

$$I = \frac{\text{charge}}{\text{period}} = q\frac{\omega}{2\pi}$$



- Magnetic moment:

$$\mu = IA = \frac{1}{2}q\omega R^2$$

- Can be expressed through angular momentum $L = m\omega R^2$

$$\vec{\mu} = -\frac{q}{2m}\vec{L}$$

For negative charge m and L are in opposite directions

- A magnetic dipole precesses about the direction of constant magnetic field

$$\left.\begin{array}{l}\dfrac{d\vec{L}}{dt} = \vec{\tau} \\ \vec{\tau} = \vec{\mu}\times\vec{B}\end{array}\right\}$$

$$\frac{d\vec{\mu}}{dt} = -\frac{q}{2m}\vec{\mu}\times\vec{B}$$

- An electron also has an intrinsic magnetic dipole moment associated with its spin S:

with g_s = 2.0023

$$\vec{\mu} = -g_s\frac{q}{2m}\vec{S}$$

Gyromagnetic ratios

Test questions

- Consider Hall effect in two samples with different carrier concentration. Which sample will show higher magnitude of Hall voltage?