Lecture contents

- **Magnetic field-2**
  - **Magnetization**
  - **Faraday's law**

Differential form of Ampere's Law

- Applying Stokes's theorem to Ampere's Law

$$\oint_C \vec{B}\cdot d\vec{l} = \int_S \nabla\times\vec{B}\cdot d\vec{s}$$

$$= \mu_0 I_{encl} = \mu_0 \int_S \vec{J}\cdot d\vec{s}$$

- Because the above must hold for any surface *S*, we must have

$$\boxed{\nabla\times\vec{B} = \mu_0 \vec{J}}$$

← Differential form of Ampere's Law

Divergence of *B*-Field

- The B-field is ***solenoidal***, i.e. the divergence of the B-field is identically equal to zero:

$$\nabla \cdot \vec{B} = 0$$

- From gauss theorem

$$\iiint_V div\vec{A}dV = \oiint_S \left(\vec{A}\cdot\vec{n}\right)ds$$

field lines are closed.

- Physically, this means that magnetic charges (monopoles) do not exist.

- A magnetic charge can be viewed as an isolated magnetic pole.

Magnetization-1

- Similar to electric polarization due to dipole moment density, magnetization is magnetic dipole density

$$\vec{M} = \frac{\sum_i \vec{\mu}_i}{V} \quad \left\{\frac{\text{A}}{\text{m}}\right\}$$

Dimensionally as surface current density

- Similar to electric polarization, "bound" currents can be introduced:

$$\vec{J}_b = \nabla \times \vec{M}$$

- In a medium with magnetization, magnetic field (induction) depends on macroscopic and bound "dipole" currents

$$\nabla \times \frac{\vec{B}}{\mu_0} = \vec{J} + \vec{J}_b \equiv \vec{J} + \nabla \times \vec{M}$$

or

$$\nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M}\right) = \vec{J}$$

$$\frac{\vec{B}}{\mu_0} - \vec{M} \rightarrow \vec{H}$$

- Similar to electric displacement *D*, "magnetic field" *H* is introduced which does not depend upon magnetization of material
- Similar to dielectrics in most cases magnetization is proportional to magnetic field. Historically, susceptibility χ is introduced in the following way:

$$\boxed{\vec{M} = \chi \vec{H}} \quad \text{or} \quad \vec{M} = \frac{\chi}{\mu_0 (1+\chi)} \vec{B}$$

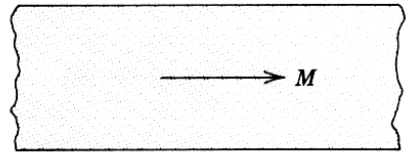
- Also relative magnetic permittivity μ_R:

$$\vec{B} = \mu_0 \mu_R \vec{H} \qquad \boxed{\mu_R = 1 + \chi}$$

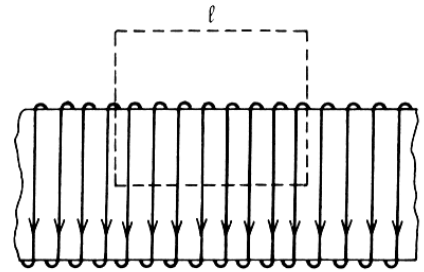
Magnetization-2

- Magnetic induction field is the same in a uniformly magnetized cylinder as in a solenoid

- Bound currents flow only along the surfaces across which magnetization change (such as surface of the cylinder).



$$B = \mu_0 M$$



$$B = \mu_0 n I$$

Magnetization equals surface current density

$$M = nI \quad \left\{ \frac{\text{A}}{\text{m}} \right\}$$

Fundamental Postulates of Magnetostatics

- Ampere's law in differential form

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

- No isolated magnetic charges

$$\nabla \cdot \vec{B} = 0$$

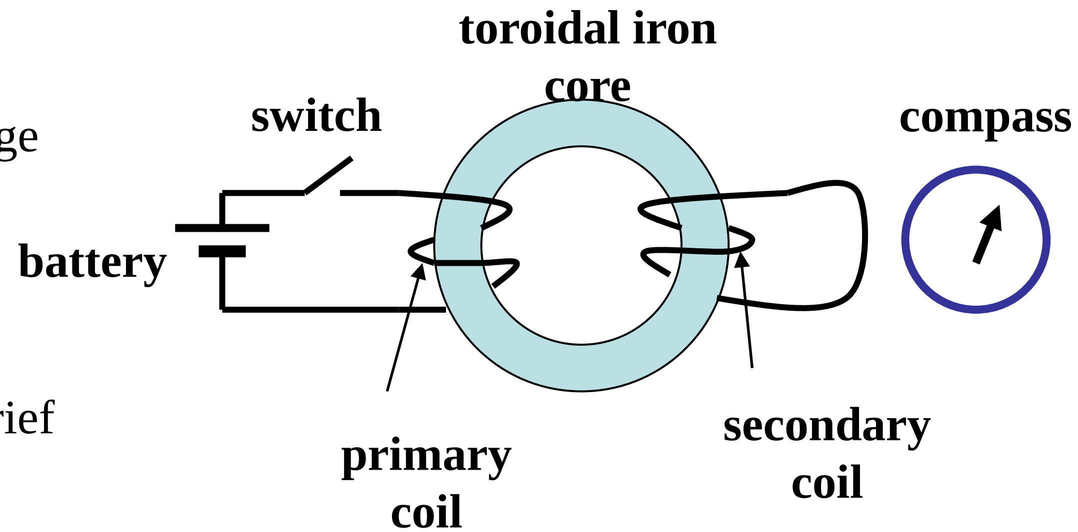
B is solenoidal

The Three Experimental Pillars of Electrodynamics

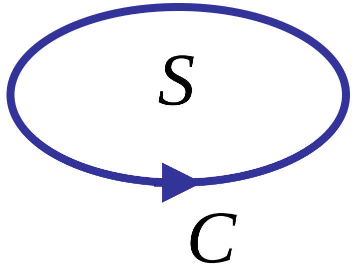
- Electric charges attract/repel each other as described by ***Coulomb's law***.
- Current-carrying wires attract/repel each other as described by ***Ampere's law of force***.
- Magnetic fields that change with time induce electromotive force as described by ***Faraday's law***.

Faraday's Experiment

- Upon closing the switch, current begins to flow in the *primary coil*.
- A momentary deflection of the *compass needle* indicates a brief surge of current flowing in the *secondary coil*.
- The *compass needle* quickly settles back to zero.
- Upon opening the switch, another brief deflection of the *compass needle* is observed.



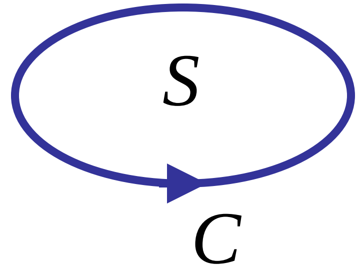
- Faraday's Law of Electromagnetic Induction: "The electromotive force induced around a closed loop ***C*** is equal to the time rate of decrease of the magnetic flux linking the loop."



$$V_{ind} = -\frac{d\Phi}{dt} \equiv -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s}$$

Faraday's Law of Electromagnetic Induction

$$V_{ind} = -\frac{d\Phi}{dt} \equiv -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s}$$



- Electromotive force = potential difference

$$V_{ind} = \oint_C \vec{E}\cdot d\vec{l}$$

- Integral form of Faraday's law

$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s}$$

- Using Stokes' theorem:

assuming a stationary surface S

$$\oint_C \vec{E}\cdot d\vec{l} = \int_S \nabla\times\vec{E}\cdot d\vec{s} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s} = -\int_S \frac{\partial\vec{B}}{\partial t}\cdot d\vec{s}$$

- Faraday law in differential form:

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

- Faraday's law states that a changing magnetic field induces an electric field.
- The induced electric field is *non-conservative*.