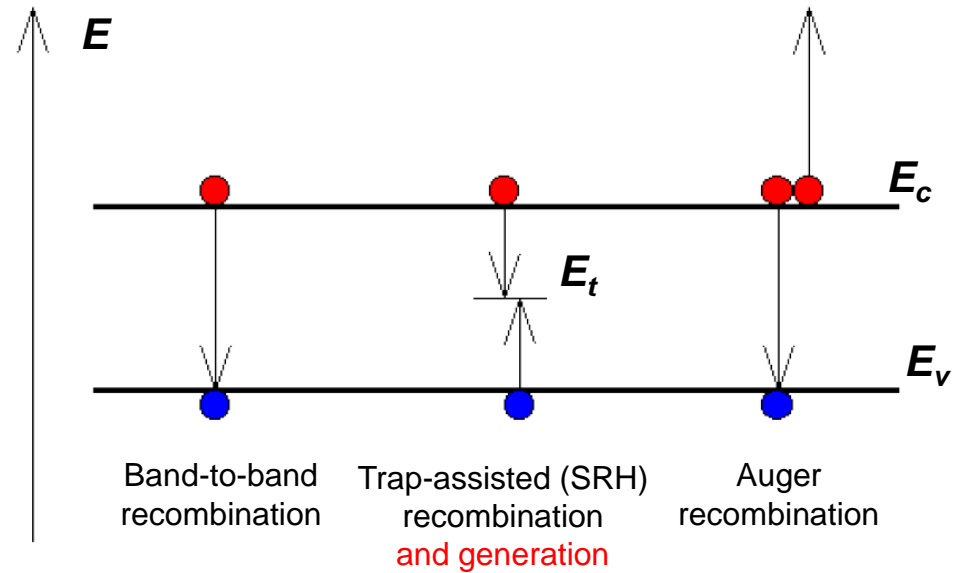
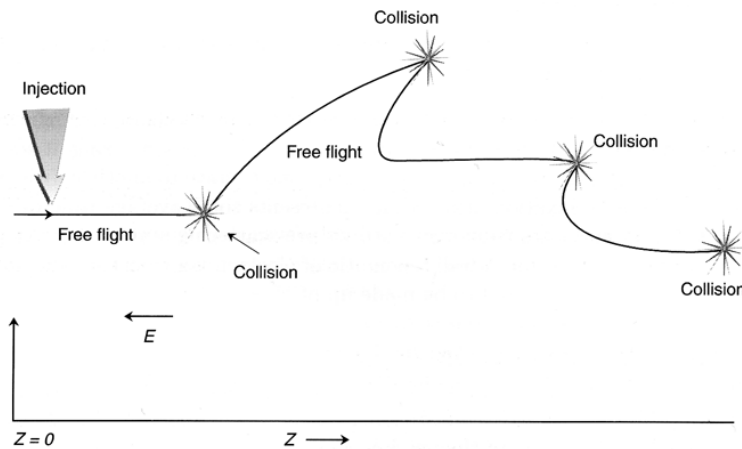


Lecture contents

- Transport, scattering
- Generation/recombination



Electron transport: General considerations

How “free” carriers react on external electric field ?

Motion in real space = thermal motion
+ drift + scattering

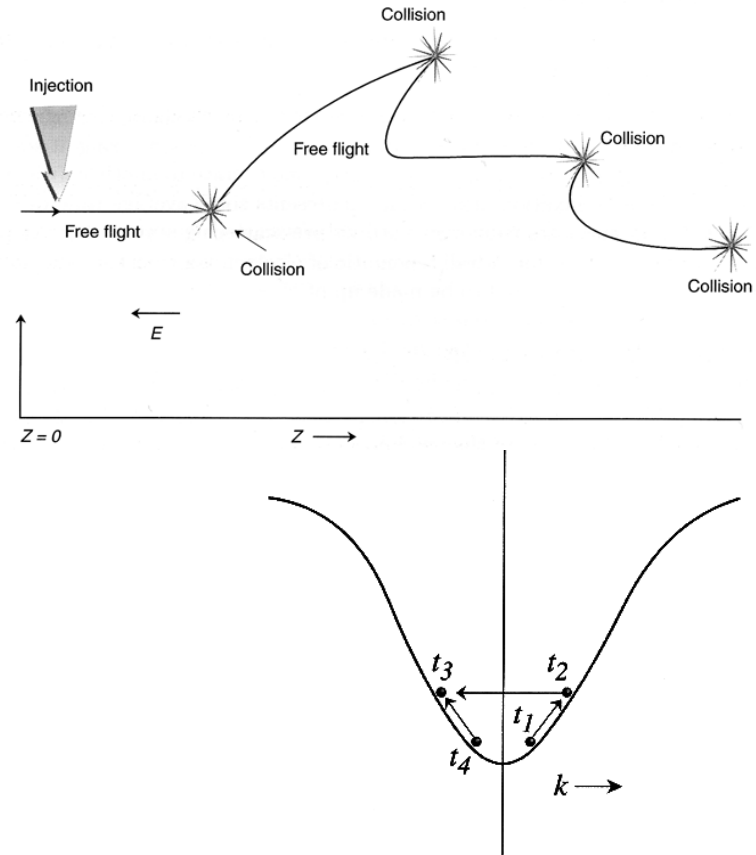
Mean free path:

$$\lambda = v_{th} \tau_m$$

Motion in wavevector space: inside a
band valley (unless intervalley
scattering is involved)

Current density is proportional to drift
velocity of carriers

$$J = env_d \left[\frac{\text{A}}{\text{cm}^2} \right]$$



Electron transport: phenomenological approach

Kinetic energy per electron in 3D (from kinetic theory of gases)

$$\frac{mv^2}{2} = \frac{3}{2} k_B T$$

Thermal velocity

$$v_{th} = \left(\frac{3k_B T}{m^*} \right)^{1/2} \sim 10^7 \frac{cm}{s} \gg v_d$$

Force on a “free” electron includes electric and “friction” with momentum relaxation time τ_m : (\mathcal{E} – electric field)

$$m_e^* \frac{dv_d}{dt} = e\mathcal{E} - m_e^* \frac{v_d}{\tau_m}$$

$$\mu = \frac{e\tau_m}{m_e^*} \left[\frac{cm^2}{Vs} \right]$$

In the steady state drift velocity is proportional to the field (μ – drift mobility):

$$|v_d| = \frac{e\tau_m}{m_e^*} |\mathcal{E}| \equiv -\mu |\mathcal{E}|$$

And current density (σ – conductivity) gives Ohm’s law:

$$J = env = en\mu\mathcal{E} \equiv \sigma\mathcal{E}$$

$$\sigma = en\mu = \frac{e^2\tau_m}{m_e^*} n$$

For semiconductor containing both electron and holes:

$$\sigma = e(n\mu_e + p\mu_h)$$

$$\sigma = \begin{cases} 3D: (\Omega \cdot cm)^{-1} \\ 2D: (\Omega/sq.)^{-1} \end{cases}$$

Scattering mechanisms

Mathiessen's rule for relaxation time:

$$\frac{1}{\tau_m} = \frac{1}{\tau_{ac}} + \frac{1}{\tau_{op}} + \frac{1}{\tau_{ii}} + \frac{1}{\tau_{ni}} + \dots$$

In low electric fields

- ionized impurities
- acoustic phonons
 - Deformation potential
 - Piezoelectric

In high electric fields

- optical phonons
- intervalley scattering

At high concentrations

- carrier-carrier scattering

To understand how these mechanisms affect mobility one needs to consider dependence $\tau(E)$

Conductivity effective mass

$$m_e^* = m_{\Gamma}$$

- For non-degenerate conduction band minimum (Γ -point):

- Indirect conduction band minimum: effective mass will depend on direction!

- For example: Si along [100] direction:
$$J_n = J_{ex} + J_{ey} + J_{ez} = \left(2 \frac{n e^2 \tau_m}{6 m_l} + 2 \cdot 2 \frac{n e^2 \tau_m}{6 m_t} \right) \mathcal{E}$$

- Conductivity effective mass of Si [100] electrons::
$$\frac{1}{m_c^*} = \frac{1}{3} \left(\frac{1}{m_l} + \frac{2}{m_t} \right)$$

- Valence band maximum: need to add up contributions of light and heavy holes:

$$J_p = J_{lh} + J_{hh} = \left(\frac{p_{lh}}{m_{lh}} + \frac{p_{hh}}{m} \right) e^2 \tau_{mh} \mathcal{E}$$

- Conductivity effective mass of holes

$$\frac{1}{m_v^*} = \frac{1}{p} \left(\frac{p_{lh}}{m_{lh}} + \frac{p_{hh}}{m_{hh}} \right) = \frac{m_{lh}^{1/2} + m_{hh}^{1/2}}{m_{lh}^{3/2} + m_{hh}^{3/2}}$$

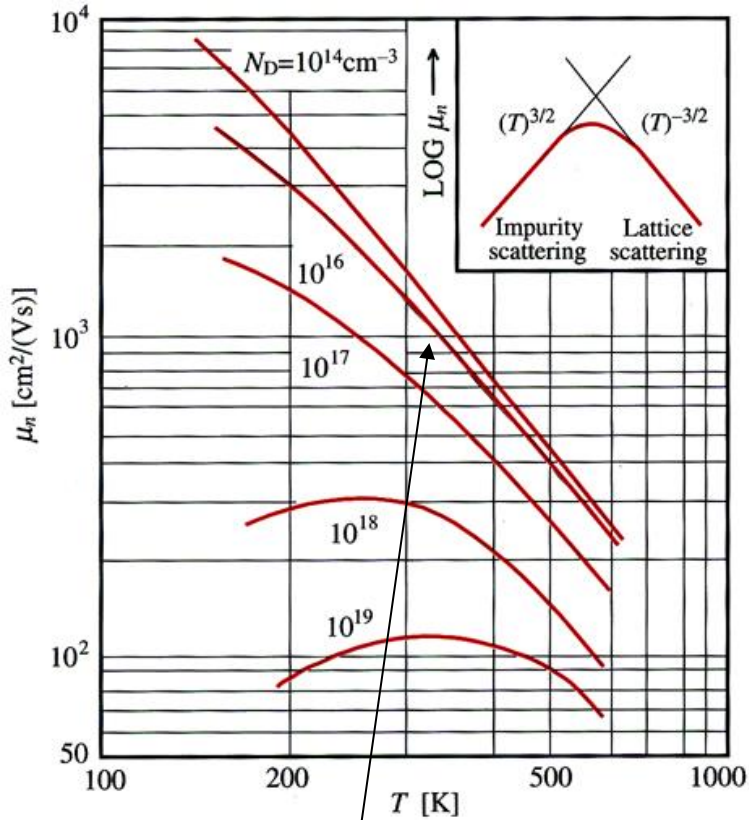
Effective masses and low field mobilities at room temperature:

	a (Å)	ϵ_r (rel.)	ρ (g/cm ⁻³)	E_g (eV)	m_n	m_p	μ_n (cm ² /Vs)	μ_p (cm ² /Vs)
Si	5.43	11.8	2.33	1.12	1.08	0.56	1350	480
Ge	5.66	16.0	5.32	0.67	0.55	0.37	3900	1900
GaAs	5.65	13.2	5.31	1.42	0.067	0.48	8500	400
InP	5.87	12.1	4.79	1.35	0.080	–	4000	100

Temperature dependence of mobility

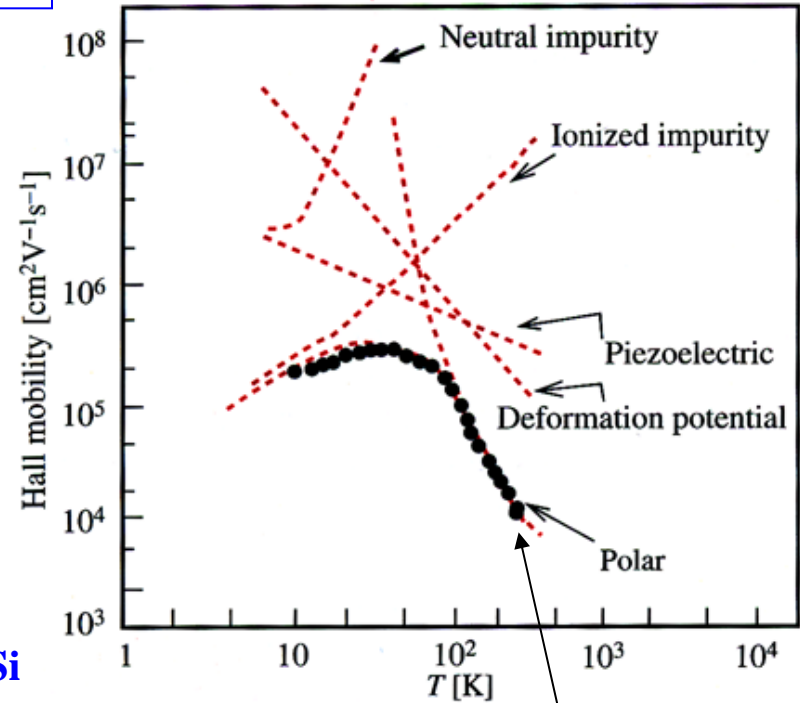
Relaxation time τ_m depends on energy, and therefore mobility depends on temperature

Mobility in n-Si



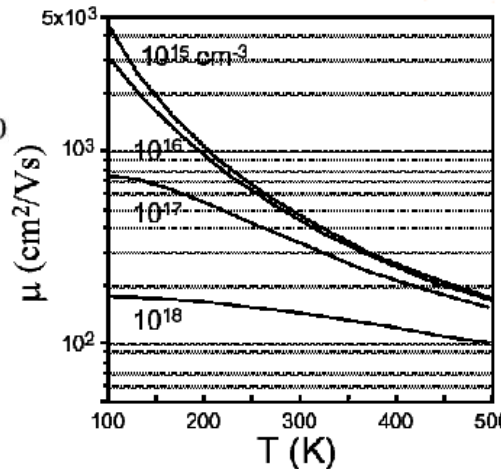
Intravalley phonon scattering + (2TA+LO)
intervalley phonon scattering

Mobility in n-GaAs



LO phonon scattering

Mobility in p-Si



From Yu and Cordona, 2003,
and Shur, 2003

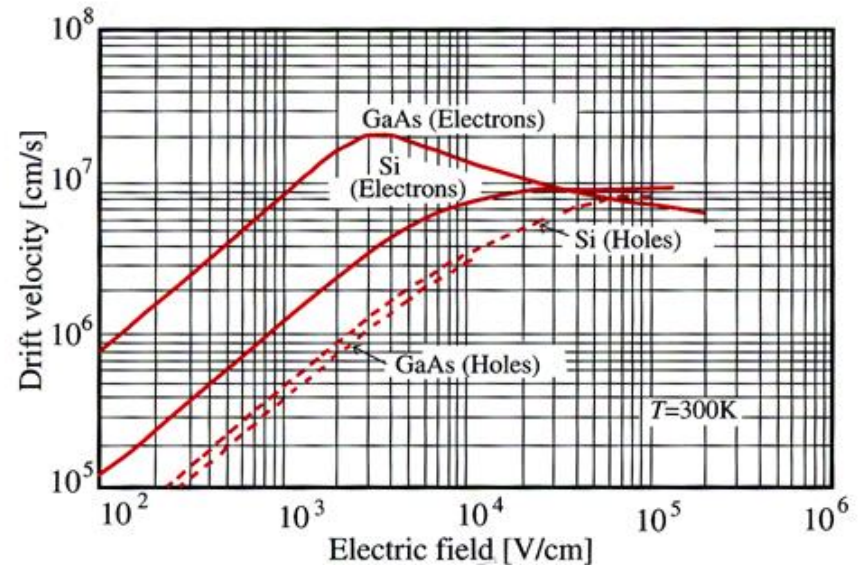
High field electron transport

Hot electrons transfer energy into thermal vibrations of the crystal lattice (phonons). Such vibrations can be modeled as harmonic oscillations with a certain frequency, ω_{ph} . The energy levels of a harmonic oscillator are equidistant with the energy difference between the levels equal to

$$E_{ph} = \hbar\omega_{ph}$$

Two types of phonons scatter electrons differently: acoustical (slow) and optical (fast):

Hence, process for a hot electron can be represented as follows. The electron accelerates in the electric field until it gains enough energy to excite optical phonon:



$$E_{ac} = \hbar\omega_{ac} \approx kT$$

$$E_{opt} = \hbar\omega_{opt} \approx 40meV$$

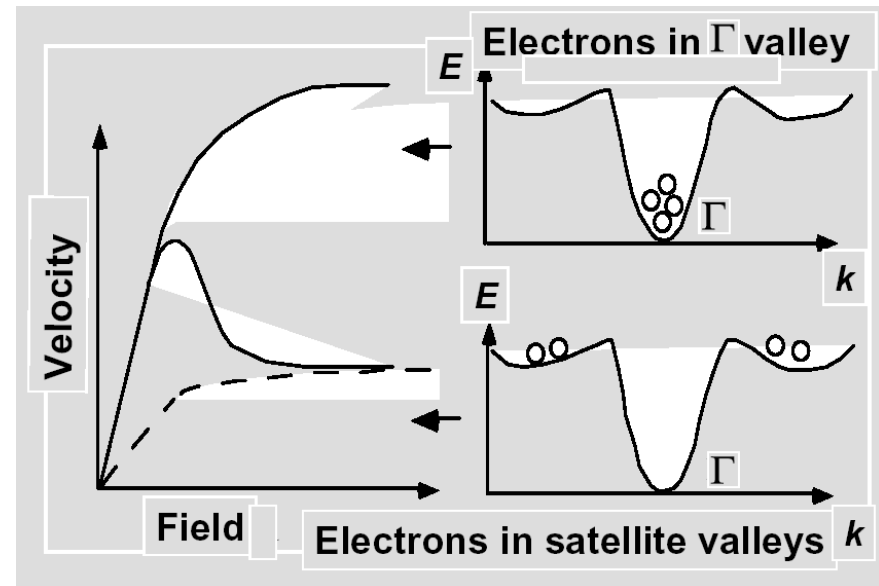
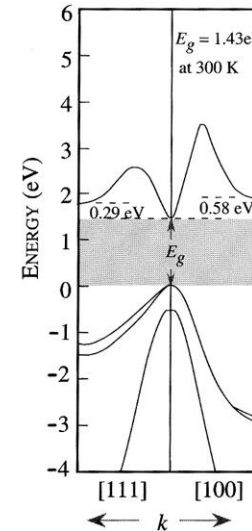
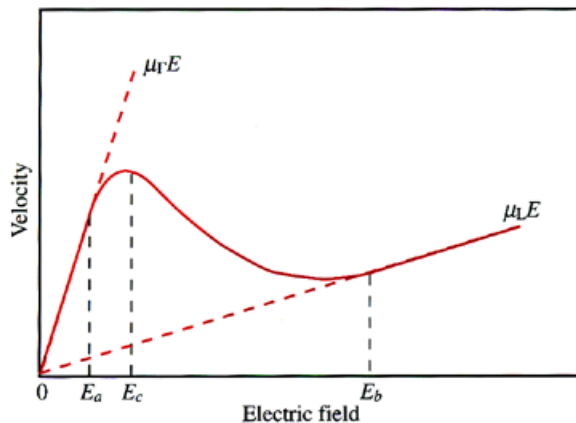
Intervalley scattering in high electric field

GaAs has an L valley just 0.29 eV higher than Γ -valley

At high fields electrons can be transferred into L-valleys. The conductivity will depend on concentrations and mobilities of electrons in both valleys:

$$\sigma = e(N_{\Gamma}\mu_{\Gamma} + N_L\mu_L)$$

Model for dependence of drift velocity on electric field in GaAs



From Shur, 2003

Diffusion and drift of carriers

- If the carrier concentration is changing from point-to point, the carriers will redistribute to equalize the concentration = Diffusion
- Diffusion is described by the first Fick's law:

$$(\text{Flux of particles}) = -(\text{Diffusion coef.}) \times (\text{gradient of concentration}) = D \cdot \nabla n$$

$$\text{Electron diffusion current} \quad J_n = eD_n \nabla n$$

$$\text{Hole diffusion current} \quad J_p = -eD_p \nabla p$$

- If electric field (weak) is applied:

$$\text{Total electron current:} \quad J_n = ne\mu_n \mathcal{E} + eD_n \nabla n$$

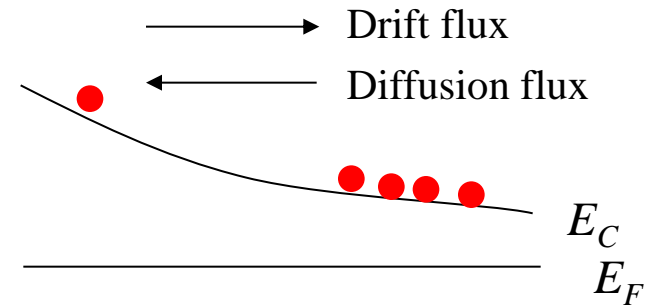
$$\text{Total hole current:} \quad J_p = pe\mu_o \mathcal{E} - eD_p \nabla p$$

$$\frac{k_B T}{e} \frac{\nabla n}{n} \text{ plays the role of an } \underline{\text{effective field}}$$

- Diffusion coefficient and diffusion current have sense only if concentration change is small on the mean free path: $|\nabla n| \lambda \ll n$

- Substituting the gradient with the effective field: $\left| \frac{e\mathcal{E}\lambda}{k_B T} \right| \ll 1$

Equilibrium of drift-diffusion: Einstein relation



At equilibrium condition (no total current)
diffusion current is equal to drift current:

$$ne\mu_n \mathcal{E} + eD_n \nabla n = 0$$

For nondegenerate semiconductor: $n = N_C \exp\left(\frac{E_F - E_C}{k_B T}\right)$

Carrier concentration is changing in the
electric field, following the potential $\varphi(r)$:

$$n(x) = n_0 \exp\left(\frac{e\varphi(x)}{k_B T}\right)$$

Gradient of electron concentration is :

$$\nabla n = n \frac{e}{k_B T} \nabla \varphi(x) = n \frac{e}{k_B T} \mathcal{E}$$

And substituting, obtain Einstein relation:

$$D = \frac{k_B T}{e} \mu$$

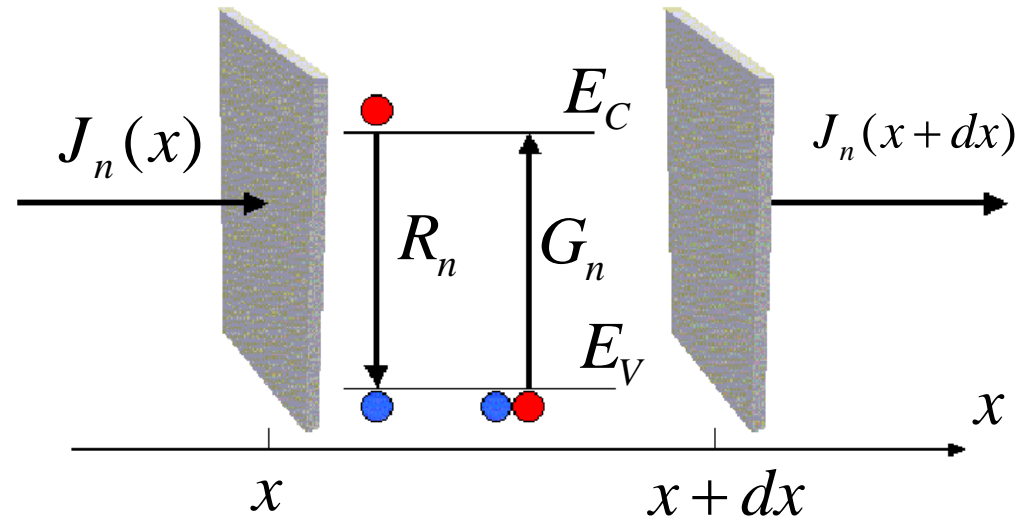
$$\mu = \frac{e\tau_m}{m_e^*}$$

Continuity equation

The continuity equation allows us to calculate the carrier distribution in the presence of generation and recombination

$$G, R = \left[\frac{1}{\text{cm}^3 \text{s}} \right]$$

The rate of change of the carriers between x and $x + dx$ is determined by fluxes and generation/recombination of carriers:



$$\frac{\partial n(x,t)}{\partial t} A dx = \underbrace{\left(\frac{J_n(x)}{-q} - \frac{J_n(x+dx)}{-q} \right)}_{-\frac{1}{q} \frac{\partial J_n(x)}{\partial x} dx} A + (G_n(x,t) - R_n(x,t)) A dx$$

Continuity equation for electrons:

$$\frac{\partial n(x,t)}{\partial t} = \frac{1}{q} \text{div} J_n(x,t) + G_n(x,t) - R_n(x,t)$$

And similar for holes

$$\frac{\partial p(x,t)}{\partial t} = -\frac{1}{q} \text{div} J_p(x,t) + G_p(x,t) - R_p(x,t)$$

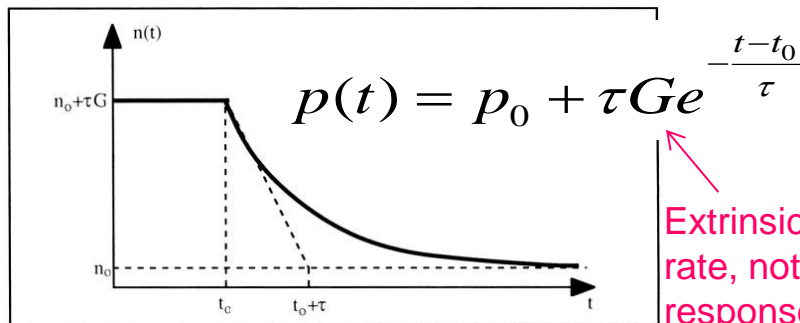
Generation/Recombination

- In the simplest model net recombination rate (intrinsic = response of the semiconductor) is proportional to the EXCESS carrier density (n_0, p_0 – equilibrium concentrations, τ 's - minority carrier lifetimes):

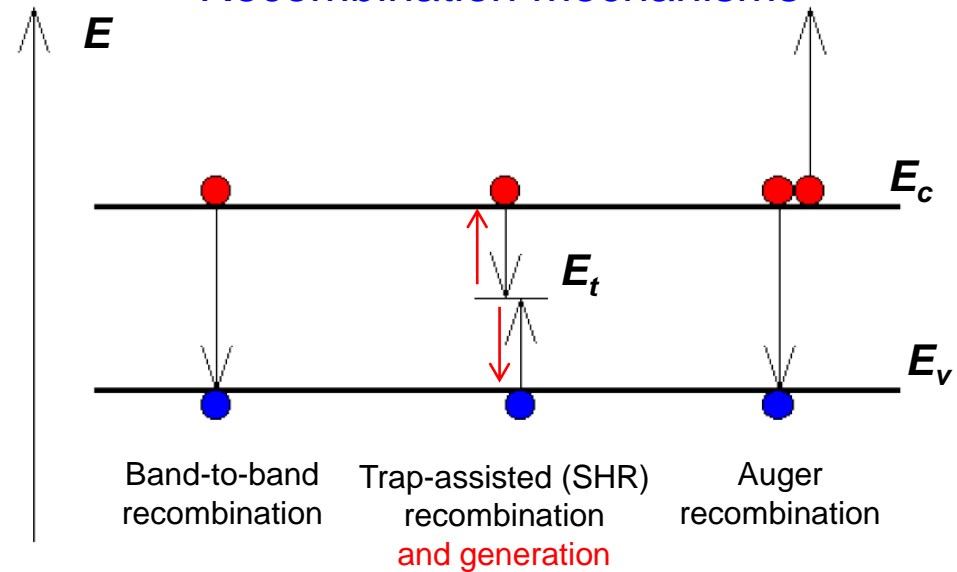
$$U_n = R_n - G_n = \frac{n - n_0}{\tau_n} \left[\frac{1}{\text{cm}^3 \text{s}} \right]$$

$$U_p = R_p - G_p = \frac{p - p_0}{\tau_p}$$

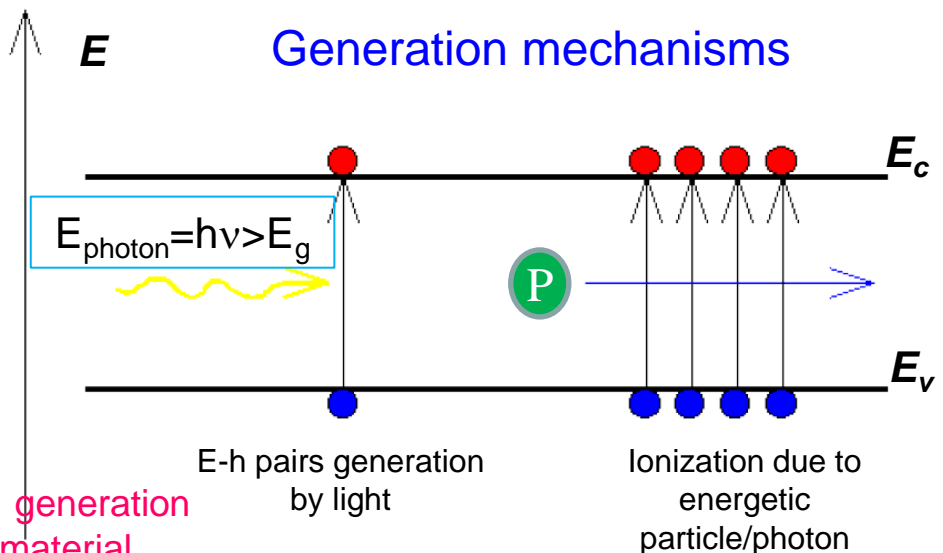
- U_n might be negative – net generation
- At thermodynamic equilibrium, the generation rate and the recombination rate are equal: $U_n = U_p = 0$
- After generation, excess concentration decays exponentially:



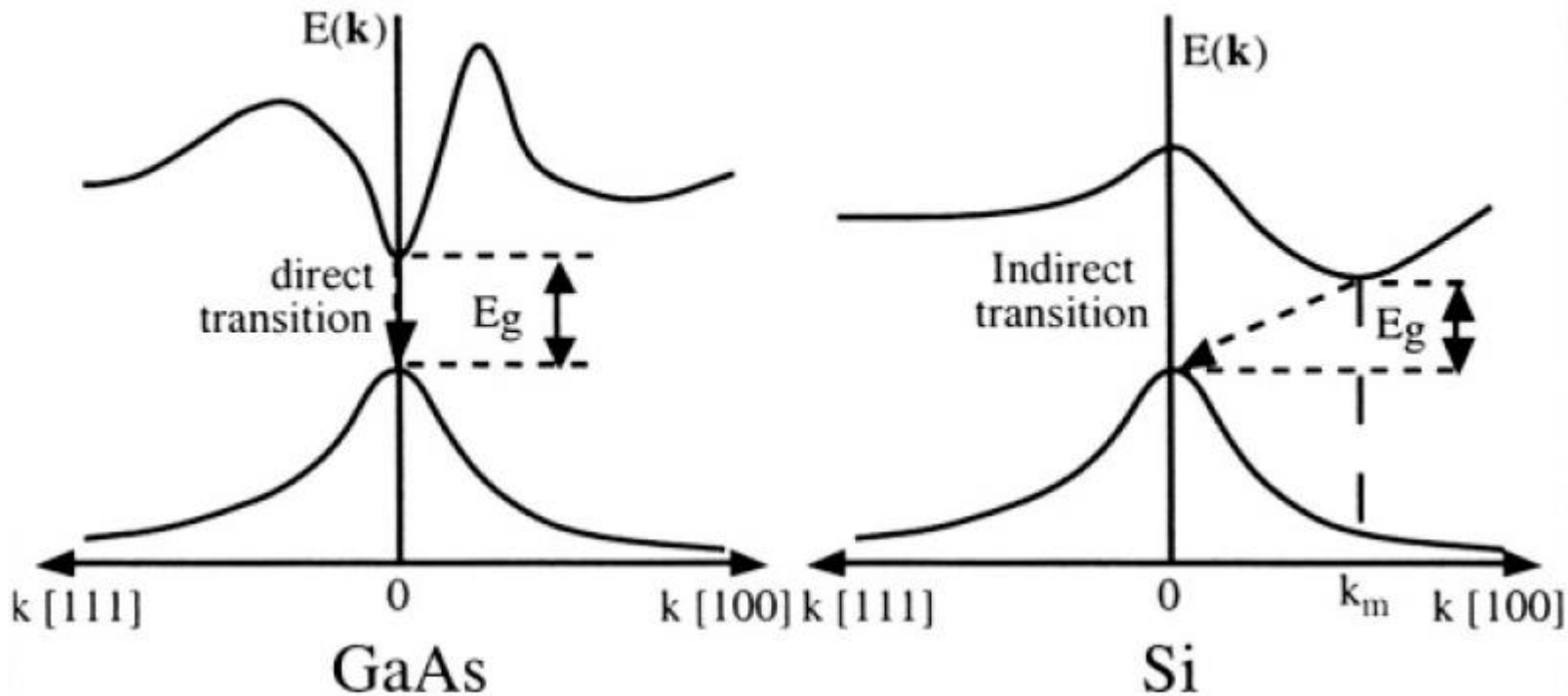
Recombination mechanisms



Generation mechanisms



Band-to-band transitions in direct-bandgap and indirect-bandgap semiconductors



- High emission probability
- Short radiative lifetime \rightarrow high radiative quantum efficiency, efficient LED, lasers...
- GaAs, InGaAs, GaN, InP, InGaSb...

- Low emission probability due to 2nd order (phonon-assisted) process (momentum conservation)
- Long radiative lifetime \rightarrow low radiative quantum efficiency
- Si, Ge, diamond, GaP, AlAs...

Optical absorption

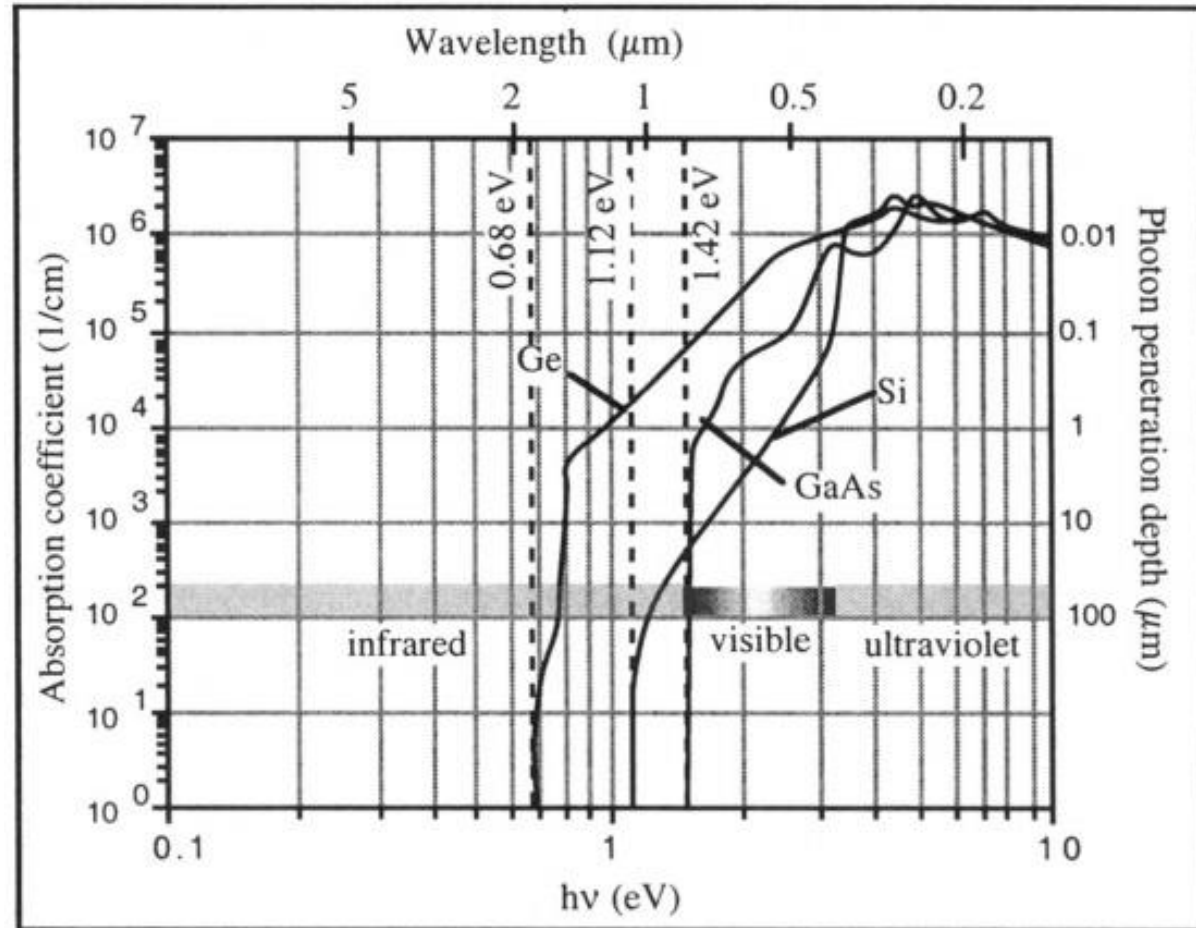
- Beer-Lambert absorption law

$$I(x) = I_0 \exp(-\alpha x)$$

- Significant absorption at

$$E_{\text{photon}} = h\nu > E_g$$

- Indirect bandgap – low near edge absorption, gradual spectral slope (momentum conservation)
- Direct bandgap – sharp edge
- Absorption spectrum depends on density of states in the bands and transition probabilities.



From Colinge, 2005

Optical absorption and non-equilibrium carriers

- Intensity [$\text{W}/\text{cm}^2 = \text{J}/\text{s}\cdot\text{cm}^2$] of light is exponentially decreasing due to absorption

$$I(x) = I_0 \exp(-\alpha x)$$

- Absorbed intensity in a layer with a thickness d :

$$I_{abs} = I_0 - I_0 e^{-\alpha d}$$

- In case of uniform generation (low absorption) $\alpha d \ll \ll 1$, can be always applied to a thin slice

$$I_{abs} = I_0 \alpha d$$

- Number of generated e-h pairs is equal to the number of photon absorbed. When generation is uniform:

$$G_n = G_p = \frac{\alpha d}{d} \frac{I_{opt}}{h\nu} = \alpha \frac{I_{opt}}{h\nu}$$

Extrinsic generation rate

- Density of both electrons and holes increase under band-to-band optical excitation by the same amount

$$\Delta n = \Delta p$$

- Higher energy photons generate “hot” carriers that relax towards the band extrema

- G/R is typically slower (10^{-9} - 10^{-5} s) than energy relaxation due to scattering (10^{-14} - 10^{-12} s)

Allows to treat electrons and holes statistically independent i.e. introduce separate **quasi-Fermi** levels for electrons and holes.

Radiative recombination

Radiative recombination rate R_r and radiative life time τ_r : $\frac{dn}{dt} = -\frac{n-n_0}{\tau_r} = -R_r \quad \left(= \frac{dp}{dt} \right)$

Radiative recombination rate R_r (B - radiative or Bi-molecular recombination coefficient)

$$R_r = Bnp = B(n_0 + \Delta n)(p_0 + \Delta p) =$$

Might depend on n

$$= B\Delta n(n_0 + p_0 + \Delta n) + Bn_i^2$$

consists of equilibrium and non-equilibrium parts:

Considering optical excitation: $\Delta n = \Delta p$

New non-equilibrium $R_r (=U_r)$

Two cases:

High injection rate (bimolecular recombination):

$$\Delta n > n_0, p_0$$

$$R_r = B(\Delta n)^2 \quad \text{and} \quad \frac{1}{\tau_r} = B\Delta n$$

Low injection:

$$\Delta n < n_0$$

$$\frac{1}{\tau_r} = B(n_0 + p_0)$$

in a doped material one of the concentrations dominates

Radiative recombination

Radiative lifetime for low injection $\Delta n < n_0$

Semiconductor	τ_{rad}	
	Intrinsic	10^{17} cm^{-3} majority carriers
Si	4.6 h	2.5 ms
Ge	0.61 s	0.15 ms
GaP		3.0 ms
GaAs	2.8 μs	0.04 μs
InAs	15 μs	0.24 μs
InSb	0.62 μs	0.12 μs

From Yu and Cordona, 2003

- Minority carrier lifetime
For example for p-material lifetime of injected electrons will be

$$\frac{1}{\tau_r} = Bp_0$$

Shockley-Hall-Reed (SHR) Recombination

Non-radiative recombination through recombination centers N_t : Shockley-Hall-Reed (SHR) processes

Consider electron G/R: n_t - centers occupied with electrons (f - Fermi distribution function)

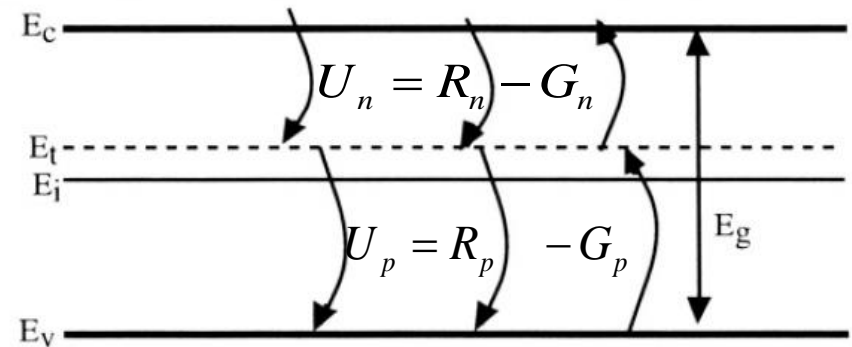
$$R_n = v_{n_th} \sigma_n n (N_t - n_t) = \frac{1}{\tau_n} (1 - f(E_t))$$

$$\text{lifetime: } \tau_n = \frac{1}{v_{n_th} \sigma_n N_t}$$

$$G_n = e_n N_t f(E_t)$$

At equilibrium, electron emission

$$e_n = v_{n_th} \sigma_n n_i \exp\left(\frac{E_i - E_t}{kT}\right)$$



Combining electrons and holes

$$U_{SHR} = \frac{np - n_i^2}{\tau_p \left(n + n_i \exp\left(\frac{E_t - E_i}{kT}\right) \right) + \tau_n \left(p + n_i \exp\left(\frac{E_i - E_t}{kT}\right) \right)}$$

for centers where the recombination rate is highest (i.e. for $E_t \approx E_i$)

$$U_{SHR} = \frac{np - n_i^2}{\tau_n (p + n_i) + \tau_p (n + n_i)}$$

For minority carriers (e.g. electrons in p-type)

$$p \approx p_0 \gg n$$

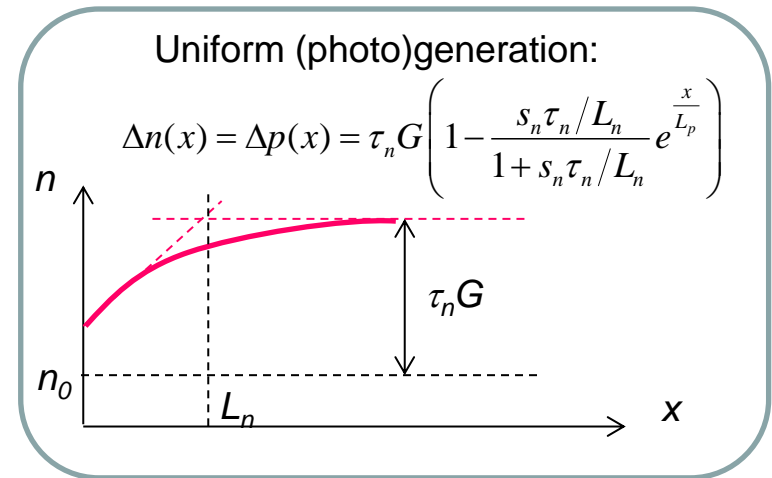
$$U_{SHR} = \frac{n - \frac{n_i^2}{p}}{\tau_n} \approx \frac{n - n_0}{\tau_n}$$

Other recombination mechanisms

Surface recombination in quasi-neutral p-type region $p \gg n, p \gg n_i$

With surface recombination velocity s_n [cm/s]

$$U_{surf,n} = s_n (n - n_0)$$



Auger recombination contribute at high carrier concentrations:

$$R_A = C_n n^2 p + C_p p^2 n \approx C n^3$$

In general in case of external generation G :

$$\frac{dn}{dt} = G - \frac{n - n_0}{\tau_n}$$

where lifetime is a result of various processes: non-radiative through recombination centers, radiative, Auger recombination:

$$\frac{1}{\tau_n} = \frac{1}{\tau_{n0}} + \frac{1}{\tau_r} + \frac{1}{\tau_A}$$

Lifetime at high excitation conditions:

$$\frac{1}{\tau_n} = A + Bn + Cn^2$$

Lecture recap

- Ohmic behavior results from scattering

- Drift-diffusion

$$J_n = ne\mu_n \mathcal{E} + eD_n \nabla n$$

- Recombination rate

$$U_n = R_n - G_n = \frac{n - n_0}{\tau_n}$$

- Continuity equation (1D)

$$\frac{\partial n}{\partial t} = \frac{1}{q} \frac{\partial J_n}{\partial x} + G_n - R_n$$

Minority carrier lifetime

