Lecture contents

- P-n junction
  - Current : Shockley model
  - Generation-recombination current
  - High injection

Currents in p-n junction

First we will use Shockley approximation:

1. Low injection = Fermi level do not change in the depletion layer = minority carrier densities << majority carrier densities = majority carrier density equals to doping concentration: $p_n \ll n_{n0} = N_d$ (in n-region and similar in p-region)
2. Non-degenerate = Boltzmann statistics
3. Drift-diffusion current (diffusion in quasi-neutral regions)
4. Long-base diode: length of the quasi-neutral regions is much larger than the diffusion length of the minority carriers L_n, L_p.
5. No generation/recombination in the depletion layer
6. Abrupt depletion layer approximation

Let' apply bias and calculate current through the p-n junction

From Sze, 1981

Shockley model: Carrier concentrations and currents ($N_d > N_a$)

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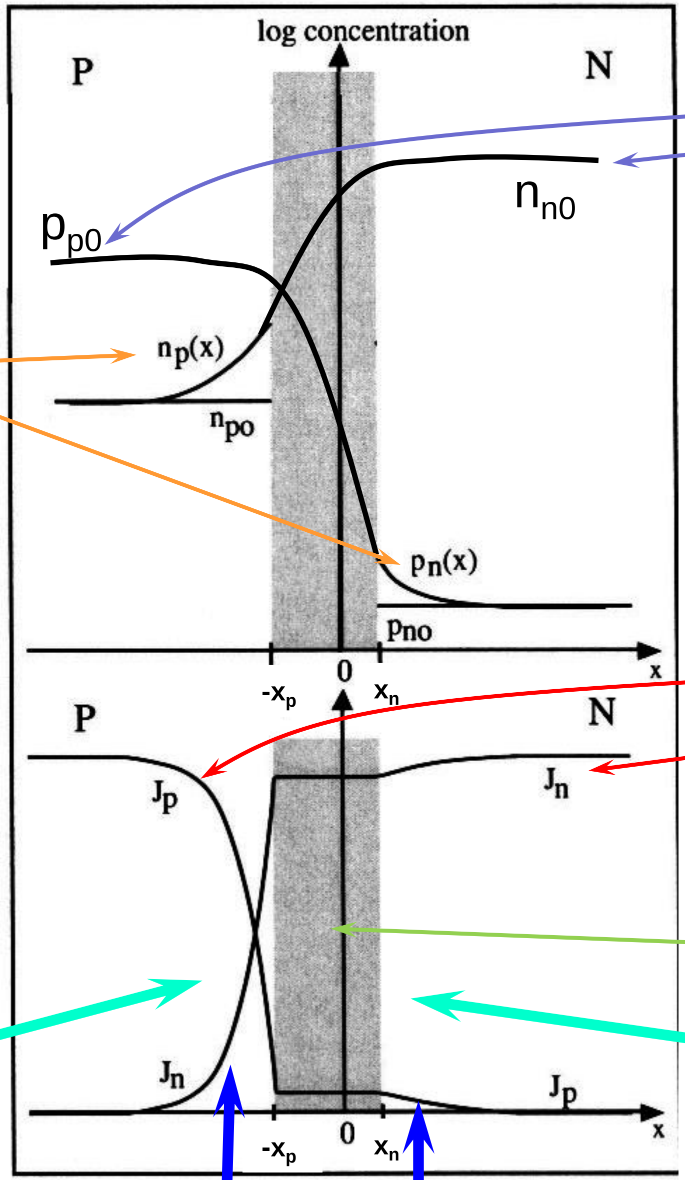


Majority carrier concentrations

Minority carrier concentrations

Majority carrier current

Total current is constant throughout the device

Depletion region

Recombination region

Recombination region

Minority carrier current

From Colinge, Colinge

Currents in p-n junction: Shockley theory

Definitions of quasi-Fermi levels:

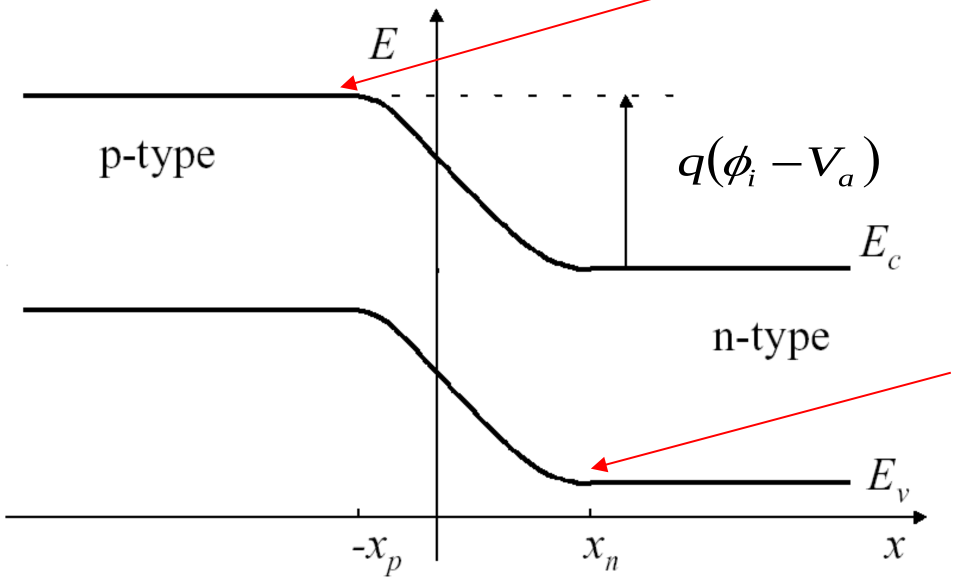
$$n = n_i \exp\left(\frac{E_{Fn} - E_i}{kT}\right) = N_c \exp\left(\frac{E_{Fn} - E_c}{kT}\right)$$

$$p = n_i \exp\left(\frac{E_i - E_{Fp}}{kT}\right) = N_v \exp\left(\frac{E_v - E_{Fp}}{kT}\right)$$

Potential across the junction: $qV_a = E_{Fp} - E_{Fn}$

Minority carrier concentrations at the edges of the depletion region (will serve as boundary conditions):

$$n_p(-x_p) = \frac{n_i^2}{N_a} e^{V_a/V_t}$$



$$p_n(x_n) = \frac{n_i^2}{N_d} e^{V_a/V_t}$$

From Sze, 1981

Currents in p-n junction

Let’s find how minority carriers currents recombine:

Continuity equation for electrons in p-region:

$$\frac{\partial n}{\partial t} = G_n - R_n + \frac{1}{q}\nabla \cdot J_n \Longrightarrow 0 = -\frac{n_p - n_{p0}}{\tau_n} + \frac{1}{q}\frac{\partial J_n}{\partial x}$$

And diffusion current (no drift in the quasi-neutral region):

$$J_n = nq\mu_n\mathcal{E} + qD_n\nabla n \Longrightarrow J_n = qD_n\frac{\partial n}{\partial x}$$

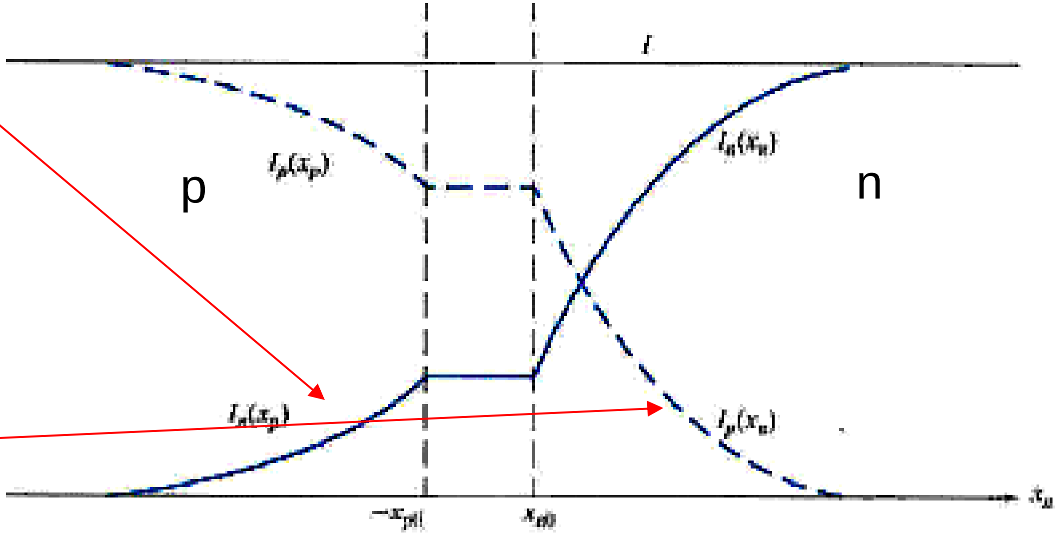
Combining:

$$D_n\frac{\partial^2 n_p}{\partial x^2} = \frac{n_p - n_{p0}}{\tau_n}$$

And similar for n-region:

$$D_p\frac{\partial^2 p_n}{\partial x^2} = \frac{p_n - p_{n0}}{\tau_p}$$

Currents



From Sze, 1981

Currents in p-n junction

Integrating minority carrier concentration

$$\frac{\partial^2 n_p}{\partial x^2} = \frac{n_p - n_{p0}}{D_n \tau_n}$$

with boundary condition

$$n_p(-x_p) = \frac{n_i^2}{N_a} e^{V_a/V_t}$$

$$n_p(x) = n_{p0} + n_{p0}\left(e^{V_a/V_t} - 1\right) e^{(x+x_p)/\sqrt{D_n \tau_n}}$$

And the minority electron current using diffusion length:

$$L_n = \sqrt{D_n \tau_n}$$

$$J_n = qD_n \frac{\partial n}{\partial x} = q\frac{D_n n_{p0}}{L_n}\left(e^{V_a/V_t} - 1\right) e^{(x+x_p)/L_n}$$

And similar for minority hole current:

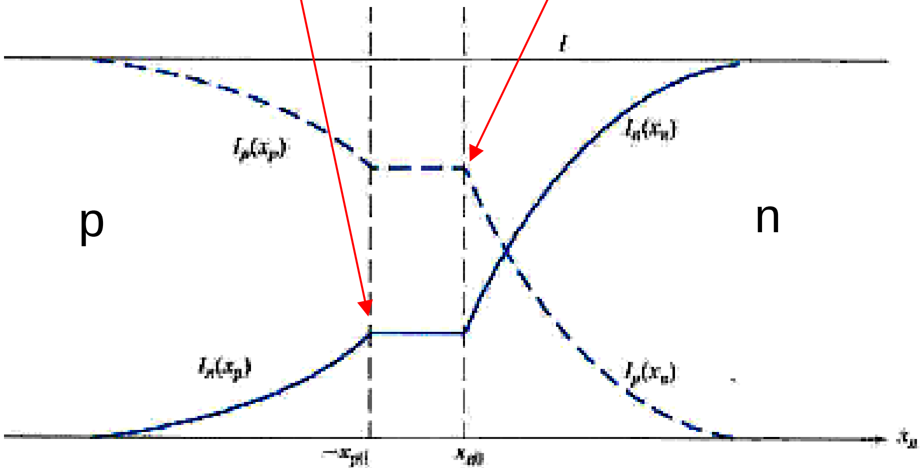
$$J_p = q\frac{D_p p_{n0}}{L_p}\left(e^{V_a/V_t} - 1\right) e^{-(x-x_n)/L_p}$$

From Sze, 1981

I-V characteristics of p-n junction: Shockley model

If we can ignore the change of current in the depletion region (no generation/recombination), we can obtain total current as a sum of minority carrier currents at respective edges of depletion region:

$$J_{tot} = J_n(-x_p) + J_p(x_n)$$



Combining:

$$\boxed{J_{tot} = J_s\left(e^{V_a/V_t} - 1\right)}$$

With saturation current:

(Mostly determined by low-doped side!)

$$\boxed{J_s = q\frac{D_p p_{n0}}{L_p} + q\frac{D_n n_{p0}}{L_n}}$$

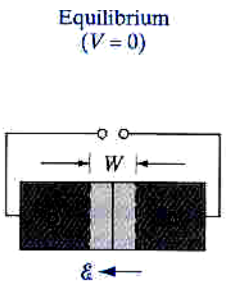
$$q\sqrt{\frac{D_n}{\tau_n}}\frac{n_i^2}{N_a}$$

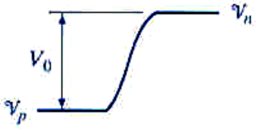
From Sze, 1981

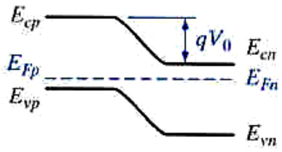
Bias and currents in p-n junction

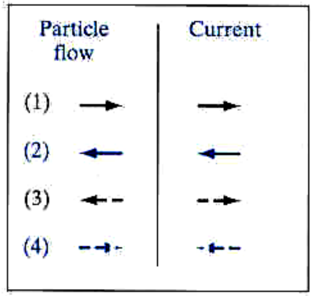
Bias | **Potential barrier at the junction** | **Energy band**

Equilibrium
V=0



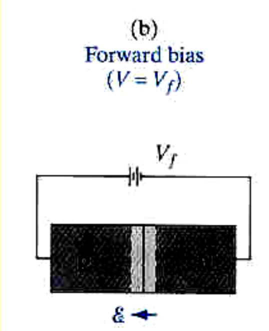


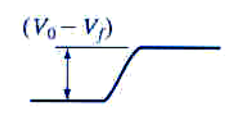


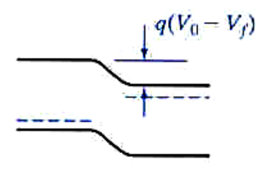


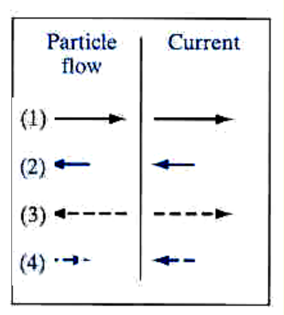
Hole diffusion
Hole drift
Electron diffusion
Electron drift

Forward bias
V_f



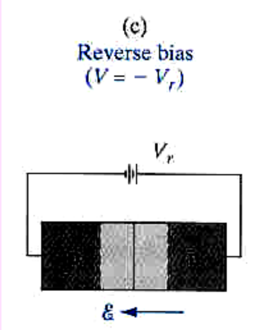


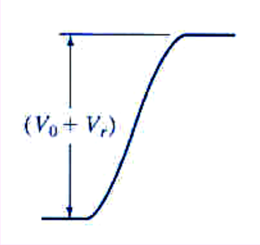


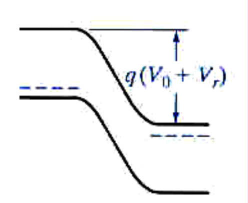


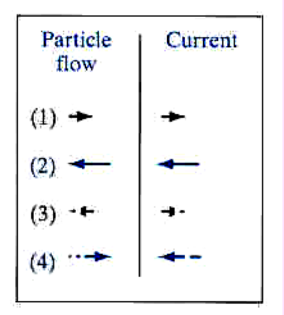
Hole diffusion
Hole drift
Electron diffusion
Electron drift

Reverse bias
V_r









Hole diffusion
Hole drift
Electron diffusion
Electron drift

Shockley model and its limitations

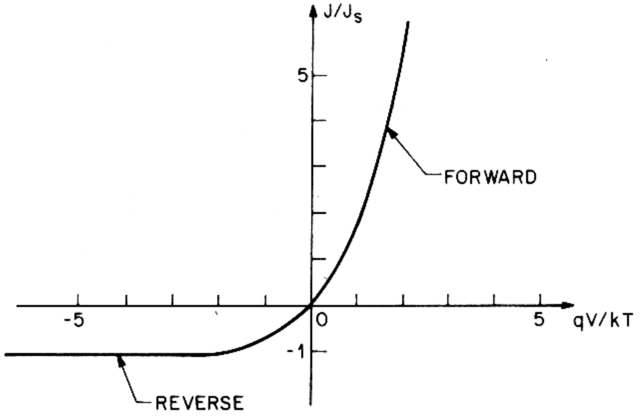
Shockley model works for **narrow-bandgap** semiconductors at **low current** densities (e.g. Ge at room temperature) when **depletion region width is much smaller than diffusion length** of minority carriers, and the **device is much longer than the diffusion lengths.**

Also the drift-difusion approximation should be valid → **low doping level at least from one side**

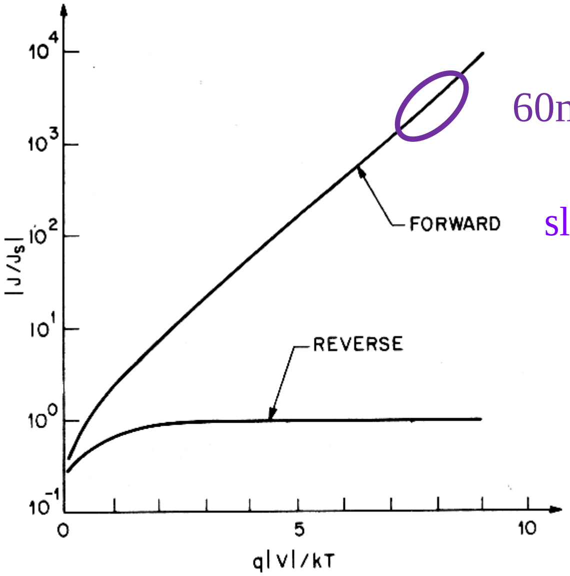
Departures from the ideal equation is due to

- Short base
- Generation-recombination in the depletion region
- Surface effects: leakage currents due to band bending
- Tunneling of carriers between states in bandgap
- Series resistance

I-V characteristics of an ideal diode



I-V characteristics of an ideal diode: semilog plot



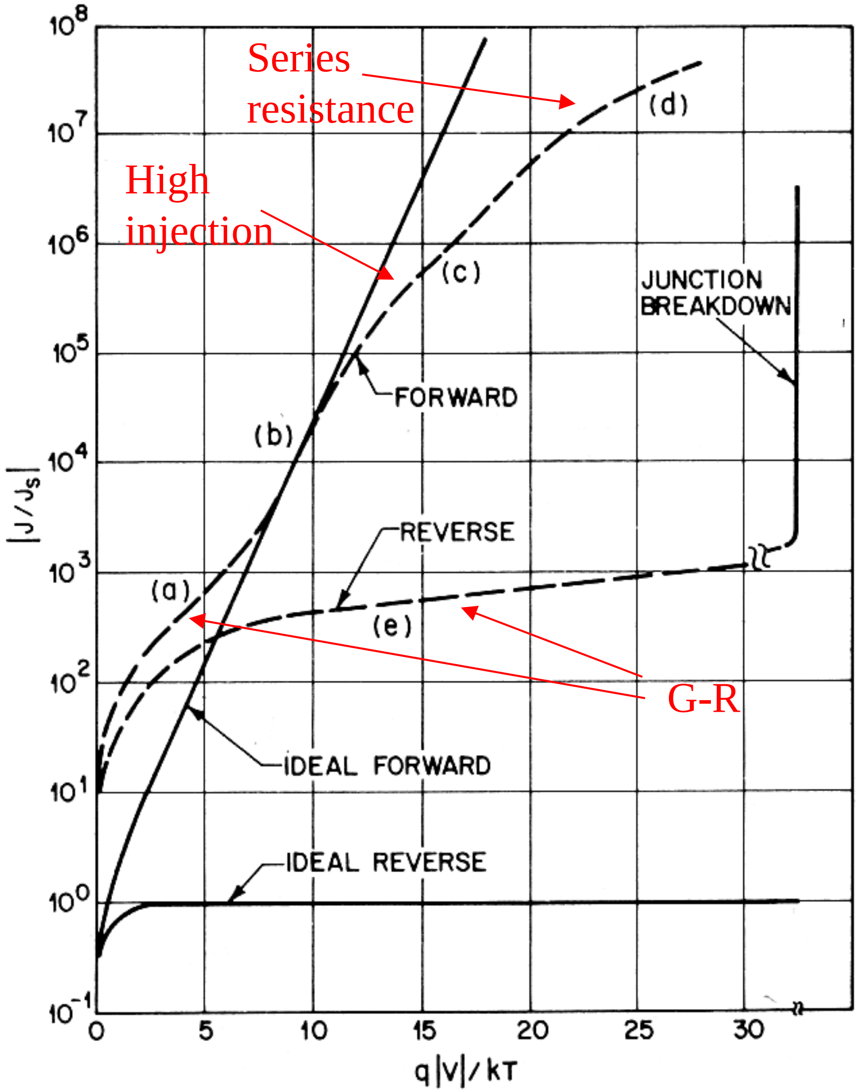
60mV/decade slope at RT:

$$\text{slope} = \frac{1}{\ln 10}\frac{q}{k_B T} = \frac{1\ \text{decade}}{60\text{mV}}$$

From Sze, 1981

Realistic p-n junction characteristics

I-V characteristics of a Si diode



Series resistance

High injection

G-R

From Sze, 1981

Generation-recombination in the depletion region

Generation/recombination in the depletion region violates one of the Shockley approximations (#5)

G-R

$$0 = G_n - R_n + \frac{1}{q}\frac{dJ_n}{dx}$$

- Radiative recombination in the depletion region:
  - is almost constant throughout the depletion region
  - Integration of the recombination rate in the depletion region increases the diode current but still has 60mV/dec. slope:

B- radiative recombination constant

$$-\Delta J_n = \Delta J_p = q\, U_{b\text{-}b}\, W = q\, W\, (np - n_i^2)\, v_{th}\, \sigma = q\, W\, n_i^2\, (e^{V_a/V_t} - 1)\, v_{th}\, \sigma$$

- Trap-assisted generation-recombination : Shockley-Hall-Reed (SHR) processes (with a trap close to intrinsic Fermi level $E_t = E_i$) :

$$U_{SHR} = \frac{np - n_i^2}{(p + n_i)\tau_{n0} + (n + n_i)\tau_{p0}}$$

  - If lifetimes are equal, the trap-assisted recombination is the highest for

$$n = p = n_i e^{V_a/2V_t} \qquad U_{SHR\max} = \frac{n_i\left(e^{V_a/2V_t} - 1\right)}{2\tau_o}$$

  - The SHR recombination current is obtained by integration of the rate over the depletion region (120 mV/dec. slope):

Effective width *x'<W*

$$\Delta J_n = -\Delta J_p = q\, U_{SHR\max}\, x' = q\, x'\, n_i\, (e^{V_a/2V_t} - 1)/2\tau_0$$

From Van Zeghbroeck, 1998

Generation-recombination in the depletion region

- Total diode current with contribution of radiation/recombination:

$$J_t = q\left(\frac{D_n n_{p0}}{L_n} + \frac{D_p p_{n0}}{L_p} + W\, n_i^2\, v_{th}\, \sigma\right)\left(e^{V_a/V_t} - 1\right) + \frac{q\, x'\, n_i}{2\tau_0}\left(e^{V_a/2V_t} - 1\right)$$

- At reverse bias the carrier concentrations in the depletion region are small:

$$np \ll n_i^2$$

    - The **generation** of carriers is the dominant process
    - The total reverse current will be higher than in Shockley model:

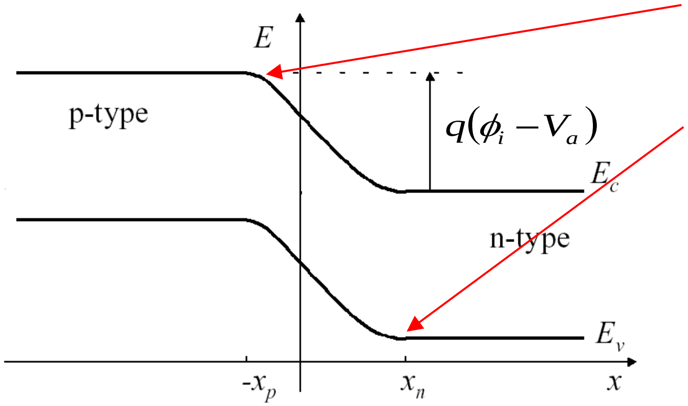
Dominant term when G-R in depletion region takes place

$$J_{reverse} = q\sqrt{\frac{D_p}{\tau_p}}\,\frac{n_i^2}{N_D} + BWn_i^2 + \frac{qn_i x'}{2\tau_0}$$

High injection (direct bias)

High injection violates one of the Shockley approximations that the majority carriers are determined by thermal equilibrium: $p_p(-x_p) = p_{p0}(-x_p) = N_a$

but they are not. If $n_p(-x_p) \approx p_{p0}(-x_p)$ neutrality should lead to $p_p(-x_p) = p_{p0}(-x_p) + n_p(-x_p)$



$$n_p\, p_p = n_i^2\, e^{V_a/V_t} \cong n_p\,(p_{p0} + n_p)$$

$$n_n\, p_n = n_i^2\, e^{V_a/V_t} \cong p_n\,(n_{n0} + p_n)$$

Solving quadratic equations:

For large direct bias

$$n_p = \frac{N_a}{2}\left(\sqrt{1 + \frac{4\, n_i^2\, e^{V_a/V_t}}{N_a^2}} - 1\right) \cong n_i\, e^{V_a/2V_t}$$

$$p_n = \frac{N_d}{2}\left(\sqrt{1 + \frac{4\, n_i^2\, e^{V_a/V_t}}{N_d^2}} - 1\right) \cong n_i\, e^{V_a/2V_t}$$

The current will be (also ignoring recombination in the depletion region):

$$J_n + J_p = q\left(\frac{D_n}{L_n} + \frac{D_p}{L_p}\right) n_i\, e^{V_a/2V_t}$$

Reduced slope, 120mV/dec.

From Van Zeghbroeck, 1998

Quasi-Fermi levels

Definitions of quasi-Fermi levels:

$$n = n_i \exp\left(\frac{E_{Fn} - E_i}{kT}\right) = N_c \exp\left(\frac{E_{Fn} - E_c}{kT}\right)$$

$$p = n_i \exp\left(\frac{E_i - E_{Fp}}{kT}\right) = N_v \exp\left(\frac{E_v - E_{Fp}}{kT}\right)$$

Or

$$\boxed{E_{Fn} = E_i + kT \ln \frac{n}{n_i}}$$

Electrostatic potential across the junction: $qV_a = E_{Fp} - E_{Fn}$

Quasi-Fermi level can be related to the drift-diffusion current density:

$$\mu_n n \frac{dE_{Fn}}{dx} = \mu_n n \frac{dE_i}{dx} + kT \frac{\mu_n n}{n} \frac{dn}{dx} = q\mu_n \left(n\mathcal{E} + \frac{kT}{q} \mu_n \frac{dn}{dx} \right) = J_n \Rightarrow \boxed{J_n = \mu_n n \frac{dE_{Fn}}{dx}}$$

Constant quasi-Fermi level implies that the electrostatic force equals the diffusion force:

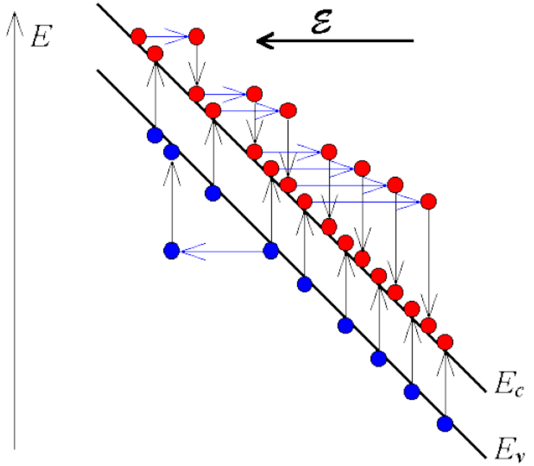
$$q\,\mathcal{E} = -\,q\,V_t \frac{d(\ln n)}{dx}$$

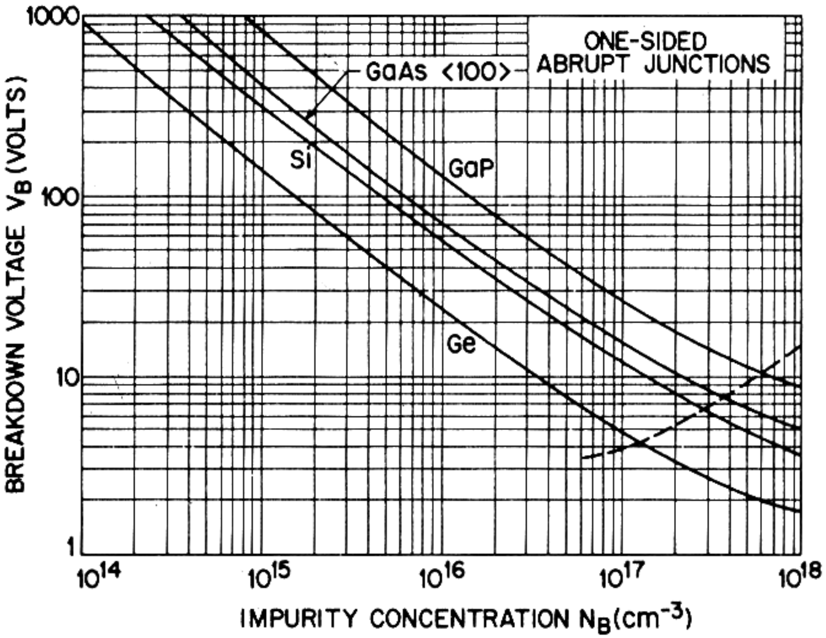
From Van Zeghbroeck, 1998

Junction breakdown

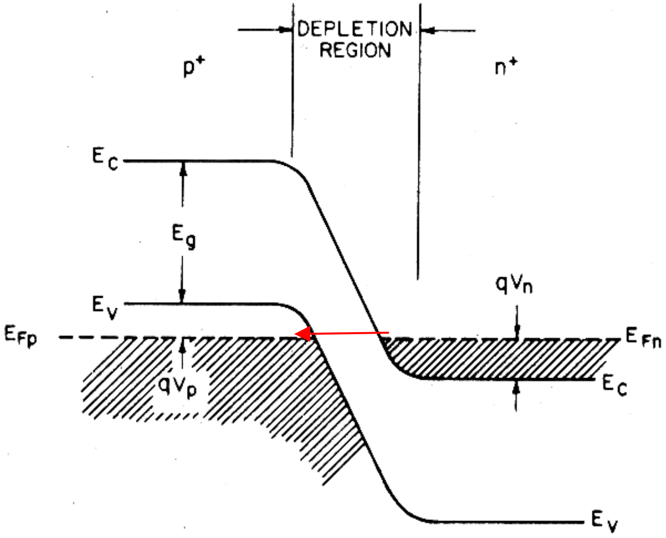
Junction breakdown occurs at high reverse bias.

- Avalanche multiplication (impact ionization of electron-hole pairs)





- Tunneling through the junction – Zener breakdown (at high doping concentrations), identical to that of tunneling in a metal-semiconductor junction.



- Thermal: junction temperature increases leading to increase of saturation current

From Van Zeghbroeck, 1998

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